

1. (10%) Compute the product  $ABC$  of the following three matrices

$$A = \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix}, B = \begin{bmatrix} 0 & 1 & 3 \\ -1 & 0 & -2 \end{bmatrix}, \text{ and } C = \begin{bmatrix} 4 \\ -1 \\ 0 \end{bmatrix}.$$

2. (10%) Simplify the following matrix expression

$$A(A + 2B) + 3B(2A - B) - A^2 + 7B^2 - 5AB$$

3. (20%) The set of solutions to a homogeneous system of linear equations is closed under addition and under scalar multiplication. It is a subspace.

4. (20%) Let  $A$  and  $B$  be matrices and  $c$  be a scalar.

1.  $(A + B)^t =$  \_\_\_\_\_ *Transpose of a sum*
2.  $(cA)^t =$  \_\_\_\_\_ *Transpose of a scalar multiple*
3.  $(AB)^t =$  \_\_\_\_\_ *Transpose of a product*
4.  $(A^t)^t =$  \_\_\_\_\_

5. (20%) Let  $A$  and  $B$  be symmetric matrices of the same size. Let  $C$  be a linear combination of  $A$  and  $B$ . Prove that  $C$  is symmetric.

6. (20%) Let  $A$  and  $B$  be symmetric matrices of the same size. Prove that the product  $AB$  is symmetric if and only if  $AB = BA$ .