1. (10 points) Let X_0 denote the initial state and P denote the transition matrix. Find the absorbing state.

$$X_0 = \begin{bmatrix} \frac{1}{3} \\ \frac{1}{3} \\ \frac{1}{3} \end{bmatrix} \quad P = \begin{array}{c} \begin{matrix} AA & Aa & aa \end{matrix} \\ \begin{bmatrix} 1 & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 1 \\ 0 & 0 & 0 \end{bmatrix} \end{array} \begin{matrix} AA \\ Aa \\ aa \end{matrix}$$

2. (10 points) Determine the minors and cofactors of the elements a_{11} and a_{32} of the following matrix A.

$$A = \begin{bmatrix} 1 & 0 & 3 \\ 4 & -1 & 2 \\ 0 & -2 & 1 \end{bmatrix}$$

3. (10 points) Evaluate the determinant of the following 4 × 4 matrix.

$$A = \begin{bmatrix} 2 & 1 & 0 & 4 \\ 0 & -1 & 0 & 2 \\ 7 & -2 & 3 & 5 \\ 0 & 1 & 0 & -3 \end{bmatrix}$$

4. (20 points) Let B be a matrix obtained by exchanging row i and row i+1 of matrix A. Let A be a square matrix. Show that det(A)=-det(B)

5. **(20 points) If A is an nxn matrix and E an nxn elementary matrix, then |EA| = |E| |A|**

6. (30 points) Let A and B are nxn matrix. If A is invertible, $A=E_1 E_2 \ldots E_n$. Show that

$|AB|= |E_1 E_2 \ldots E_n B|=|E_1 E_2 \ldots E_n||B|=|A||B|$. Show that $|A^{-1}|=1/|A|$