1. Prove the following theorems

   A. (20points)Let A be a square matrix with $|A| \neq 0$, then A is invertible with

   $$A^{-1} = \frac{1}{|A|} adj(A)$$

   B. (20points) A square matrix A is invertible if and only if $|A| \neq 0$.

   C. (10points) Let $AX = B$ be a system of n linear equations in n variables. If $|A| \neq 0$, there is a unique solution.

   D. (10points) Let $AX = B$ be a system of n linear equations in n variables. If $|A| = 0$ there may be many or no solutions.

2. (20points) Determine values of λ for which the following system of equations has nontrivial solutions. Find the solutions for each value of λ.

   $$(\lambda + 2)x_1 + (\lambda + 4)x_2 = 0$$
   $$2x_1 + (\lambda + 1)x_2 = 0$$

3. (20points) Find the eigenvalues and eigenvectors of the matrix

   $$A = \begin{bmatrix} -4 & -6 \\ 3 & 5 \end{bmatrix}$$