

1. (10 points) Compute the product ABC of the following three matrices

$$A = \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix}, B = \begin{bmatrix} 0 & 1 & 3 \\ -1 & 0 & -2 \end{bmatrix}, \text{ and } C = \begin{bmatrix} 4 \\ -1 \\ 0 \end{bmatrix}.$$

2. (10 points) Simplify the following matrix expression

$$A(A + 2B) + 3B(2A - B) - A^2 + 7B^2 - 5AB$$

3. (10 points) The set of solutions to a homogeneous system of linear equations is closed under addition and under scalar multiplication. It is a subspace.

4. (10 points) Let A and B be matrices and c be a scalar.

A. $(A + B)^t = \underline{\hspace{2cm}}$

B. $(cA)^t = \underline{\hspace{2cm}}$

C. $(AB)^t = \underline{\hspace{2cm}}$

D. $(A^t)^t = \underline{\hspace{2cm}}$

5. (10 points) Let A and B be symmetric matrices of the same size. Let C be a linear combination of A and B . Prove that C is symmetric.

6. (10 points) Let A and B be symmetric matrices of the same size. Prove that the product AB is symmetric if and only if $AB = BA$.

7. (10 points) Give the definition of inverse

8. (10 points) If a matrix has an inverse, that inverse is unique. Prove it

9. (10 points) State how to determine the inverse of a matrix.

10. (20 points) Inverse properties

A. $(A^{-1})^{-1} = \underline{\hspace{2cm}}$

B. $(cA)^{-1} = \underline{\hspace{2cm}}$

C. $(AB)^{-1} = \underline{\hspace{2cm}}$

D. $(A^n)^{-1} = \underline{\hspace{2cm}}$

E. $(A^t)^{-1} = \underline{\hspace{2cm}}$