1. Consider an economy consisting of three industries having the following input-output matrix A. Determine the output levels required of the industries to meet the demands of the other industries and of the open sector in each case.

$$A = \begin{bmatrix} 0.2 & 0.2 & 0.4 \\ 0.6 & 0.6 & 0 \\ 0 & 0 & 0.2 \end{bmatrix} \quad D = \begin{bmatrix} 9 \\ 12 \\ 16 \end{bmatrix}, \begin{bmatrix} 6 \\ 9 \\ 8 \end{bmatrix}, \text{ and } \begin{bmatrix} 12 \\ 18 \\ 32 \end{bmatrix} \text{ in turn}$$

2. Let the initial state and transition probabilities be

$$X_0 = \begin{bmatrix} 80 \\ 175 \end{bmatrix}, \quad P = \begin{array}{c} \text{(from)} \quad \text{(to)} \\ \begin{array}{cc} \text{city} & \text{suburb} \end{array} \\ \begin{bmatrix} 0.96 & 0.01 \\ 0.04 & 0.09 \end{bmatrix} \end{array} \begin{array}{l} \\ \\ \text{city} \\ \text{suburb} \end{array}$$

respectively. Find X_1, X_2, X_3, X_4 by matlab and X_n for n=20.

3. Let X_0 denote the initial state and P denote the transition matrix. Find the absorbing state by matlab.

$$X_0 = \begin{bmatrix} \frac{1}{3} \\ \frac{1}{3} \\ \frac{1}{3} \end{bmatrix} \quad P = \begin{array}{c} \begin{array}{ccc} AA & Aa & aa \end{array} \\ \begin{bmatrix} 1 & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 1 \\ 0 & 0 & 0 \end{bmatrix} \end{array} \begin{array}{l} \\ AA \\ Aa \\ aa \end{array}$$

4. If A is the adjacency matrix of a digraph. Let $a_{ij}^{(m)}$ be the element in row i and column j of A^m. Prove the number of m-paths from P_i to $P_j = (a_{ij})^{(m)}$

5. Let A be nxn. Give the cofactor expansion of A.

6. Determine the minors and cofactors of the elements a_{11} and a_{32} of the following matrix A.

$$A = \begin{bmatrix} 1 & 0 & 3 \\ 4 & -1 & 2 \\ 0 & -2 & 1 \end{bmatrix}$$

7. Evaluate the determinant of the following 4×4 matrix.

$$A = \begin{bmatrix} 2 & 1 & 0 & 4 \\ 0 & -1 & 0 & 2 \\ 7 & -2 & 3 & 5 \\ 0 & 1 & 0 & -3 \end{bmatrix}$$

8. If a matrix B is obtained from A by multiplying the elements of a row (column) by c then $|B| = c|A|$.
9. Let B be a matrix obtained by exchanging row i and row i+1 of matrix A. Let A be a square matrix. Show that det(A)=-det(B)
10. **If A is an nxn matrix and E an nxn elementary matrix, then $|EA| = |E|\,|A|$**
11. (30 points) Let A and B are nxn matrix. If A is invertible, $A=E_1 E_2 \ldots E_n$. Show that
    $|AB|= |E_1 E_2 \ldots E_nB|=|E_1 E_2 \ldots E_n||B|=|A||B|$. Show that $|A^{-1}|=1/|A|$