1. Let B be a matrix obtained by exchanging row i and row i+1 of matrix A. Let A be a square matrix. Show that det(A)=-det(B)
2. If A is an nxn matrix and E an nxn elementary matrix, then $|EA| = |E|\,|A|$
3. Let A and B are nxn matrix. If A is invertible, $A=E_1 E_2 \ldots E_n$. Show that $|AB|= |E_1 E_2 \ldots E_n B|=|E_1 E_2 \ldots E_n||B|=|A||B|$. Show that $|A^{-1}|=1/|A|$
4. Prove the following theorems
   A. Let A be a square matrix with $|A| \neq 0$, then A is invertible with

$$A^{-1} = \frac{1}{|A|} adj(A)$$

   B. A square matrix A is invertible if and only if $|A| \neq 0$.

   C. Let $AX = B$ be a system of n linear equations in n variables. If $|A| \neq 0$, there is a unique solution.

   D. Let $AX = B$ be a system of n linear equations in n variables. If $|A| = 0$ there may be many or no solutions.

5. Determine values of λ for which the following system of equations has nontrivial solutions. Find the solutions for each value of λ.

$$(\lambda+2)x_1 + (\lambda+4)x_2 = 0$$
$$2x_1 + (\lambda+1)x_2 = 0$$

6. Find the eigenvalues and eigenvectors of the matrix

$$A = \begin{bmatrix} -4 & -6 \\ 3 & 5 \end{bmatrix}$$