1. (20 points) Consider the following initial state and transition matrix.

$$\mathbf{x}_0 = \begin{bmatrix} 80 \\ 175 \end{bmatrix} \begin{matrix} \text{city} \\ \text{suburb} \end{matrix} \quad (\text{Initial populations}), \qquad P = \begin{bmatrix} 0.96 & 0.01 \\ 0.04 & 0.99 \end{bmatrix} \begin{matrix} \text{city} \\ \text{suburb} \end{matrix}$$

(from) columns: city, suburb; (to) rows: city, suburb

   A. Find the eigenvectors of P corresponding to $\lambda = 1$

   B. Assume that there is no total annual population change over the years. Find the steady state.

   C. Determine the **long-term transition matrix**

2. (20 points) Consider the following transition matrix

$$P = \begin{bmatrix} 0.6 & 0.13 \\ 0.4 & 0.87 \end{bmatrix} \begin{matrix} \text{wet} \\ \text{dry} \end{matrix} \quad (\text{following day})$$

(A given day) columns: wet, dry

   A. Determine the eigenvectors of P corresponding to $\lambda = 1$

   B. Determine the long-term transition matrix Q

3. (20 points) A **vector space** is a set V of elements called **vectors,** having operations of addition and scalar multiplication defined on it. Let **u**, **v**, and **w** be arbitrary elements of V, and c and d be scalars.

   A. State **Closure Axioms.** **B. State** addition axioms. C. **State the** scalar multiplication axioms.

4. (20 points) Let V be the set of all functions having the real numbers as their domains. Verify axioms 1, 2, 5 and 6 for vector spaces of functions

5. (20 points) Let P_n denote the set of real polynomial functions of degree $\leq n$. Prove that P_n is a vector space if addition and scalar multiplication are defined on polynomials in a pointwise manner.