1. Determine values of λ for which the following system of equations has nontrivial solutions. Find the solutions for each value of λ.

$$(\lambda+2)x_1+(\lambda+4)x_2=0$$
$$2x_1+(\lambda+1)x_2=0$$

2. Find the eigenvalues and eigenvectors of the matrix

$$A=\begin{bmatrix} -4 & -6 \\ 3 & 5 \end{bmatrix}$$

3. Consider the following initial state and transition matrix.

$$\underset{\text{Initial populations}}{\mathbf{x}_0=\begin{bmatrix} 80 \\ 175 \end{bmatrix}}\begin{matrix}\text{city}\\ \text{suburb}\end{matrix}, \quad P=\overset{\text{(from)}}{\overset{\text{city}\quad\text{suburb}}{\begin{bmatrix} 0.96 & 0.01 \\ 0.04 & 0.99 \end{bmatrix}}}\begin{matrix}\text{(to)}\\ \text{city}\\ \text{suburb}\end{matrix}$$

A. Find the eigenvectors of P corresponding to $\lambda = 1$

B. Assume that there is no total annual population change over the years. Find the steady state.

C. Determine the **long-term transition matrix**

4. Consider the following transition matrix

$$P=\overset{\text{(A given day)}}{\overset{\text{wet}\quad\text{dry}}{\begin{bmatrix} 0.6 & 0.13 \\ 0.4 & 0.87 \end{bmatrix}}}\begin{matrix}\text{wet}\\ \text{dry}\end{matrix}\quad \text{(following day)}$$

A. Determine the eigenvectors of P corresponding to $\lambda = 1$

B. Determine the long-term transition matrix Q

5. A **vector space** is a set V of elements called **vectors,** having operations of addition and scalar multiplication defined on it. Let **u**, **v**, and **w** be arbitrary elements of V, and c and d be scalars.

A. State **Closure Axioms.** **B. State** addition axioms. C. **State the** scalar multiplication axioms.

6. Let V be the set of all functions having the real numbers as their domains. Verify axioms 1, 2, 5 and 6 for vector spaces of functions

7. Let P_n denote the set of real polynomial functions of degree $\leq n$. Prove that P_n is a vector space if addition and scalar multiplication are defined on polynomials in a pointwise manner.