1. (10%) Let $\mathbf{v}_1, ..., \mathbf{v}_m$ be vectors in a vector space V. Let U be the set consisting of all linear combinations of $\mathbf{v}_1, ..., \mathbf{v}_m$. Then U is a subspace of V spanned by the vectors $\mathbf{v}_1, ..., \mathbf{v}_m$. U is said to be the vector space *generated* by $\mathbf{v}_1, ..., \mathbf{v}_m$. It is denoted $\text{Span}\{\mathbf{v}_1, ..., \mathbf{v}_m\}$

2. (10%) A set consisting of two or more vectors in a vector space is linearly dependent if and only if it is possible to express one of the vectors as a linear combination of the other vectors.

3. (10%) Let V be a vector space. Any set of vectors in V that contains the zero vector is linearly dependent.

4. (10%) Let the set $\{\mathbf{v}_1, ..., \mathbf{v}_m\}$ be linearly dependent in a vector space V. Any set of vectors in V that contains these vectors will also be linearly dependent.

5. (20%) Let the vectors $\mathbf{v}_1, ..., \mathbf{v}_n$ span a vector space V. Each vector in V can be expressed uniquely as a linear combination of these vectors if and only if the vectors are linearly independent.

6. (10%) Let $\{\mathbf{v}_1,...,\mathbf{v}_n\}$ be a basis for a vector space V. If $\{\mathbf{w}_1,...,\mathbf{w}_m\}$ is a set of more than n vectors in V, then this set is linearly dependent.

7. (10%) All bases for a vector space V have the same number of vectors.

8. (20%) Let V be a vector space of dimension n. Let $\{\mathbf{v}_1, \ldots, \mathbf{v}_m\}$ be a set of m linearly independent vectors in V, where $m < n$. Then there exist vectors $\mathbf{v}_{m+1}, \ldots, \mathbf{v}_n$ such that $\{\mathbf{v}_1, \ldots, \mathbf{v}_m, \mathbf{v}_{m+1}, \ldots, \mathbf{v}_n\}$ is a basis of V.