1. (10%) Prove that an orthogonal set of nonzero vectors in a vector space is linearly independent.
2. (10%) Let $\{\mathbf{u}_1, \ldots, \mathbf{u}_n\}$ be an orthonormal basis for a vector space $\boldsymbol{V}$. Let $\mathbf{v}$ be a vector in V. $\mathbf{v}$ can be written as a linearly combination of these basis vectors as follows:

$$\mathbf{v} = (\mathbf{v}\cdot\mathbf{u}_1)\mathbf{u}_1 + (\mathbf{v}\cdot\mathbf{u}_2)\mathbf{u}_2 + \cdots + (\mathbf{v}\cdot\mathbf{u}_n)\mathbf{u}_n$$

3. (20%) Let A be a square matrix. The following three statements are equivalent.

(a) A is orthogonal.

(b) The column vectors of A form an orthonormal set.

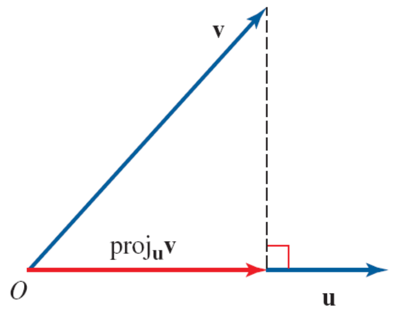
(c) The row vectors of A form an orthonormal set.

4. (10%) Let A be an orthogonal matrix. Then

(a) $|A| = \pm 1$

(b) A^{-1} is an orthonormal matrix.

5. (10%) The **projection** of a vector $\mathbf{v}$ onto a nonzero vector $\mathbf{u}$ in $\mathbf{R}^n$ is denoted $\text{proj}_{\mathbf{u}}\mathbf{v}$ and is defined by ____



$$\text{proj}_{\mathbf{u}}\mathbf{v} = \frac{\mathbf{v}\cdot\mathbf{u}}{\mathbf{u}\cdot\mathbf{u}}\mathbf{u}$$

6. (10%) Determine the projection of the vector $\mathbf{v} = (6, 7)$ onto the vector $\mathbf{u} = (1, 4)$.
7. **(20%) State the Gram-Schidt Orthogonalization Process**
8. (10%) Find a QR factorization of

$$A = \begin{bmatrix} 1 & 4 & 8 \\ 2 & 0 & 1 \\ 0 & 5 & 5 \\ 3 & 8 & 6 \end{bmatrix}.$$