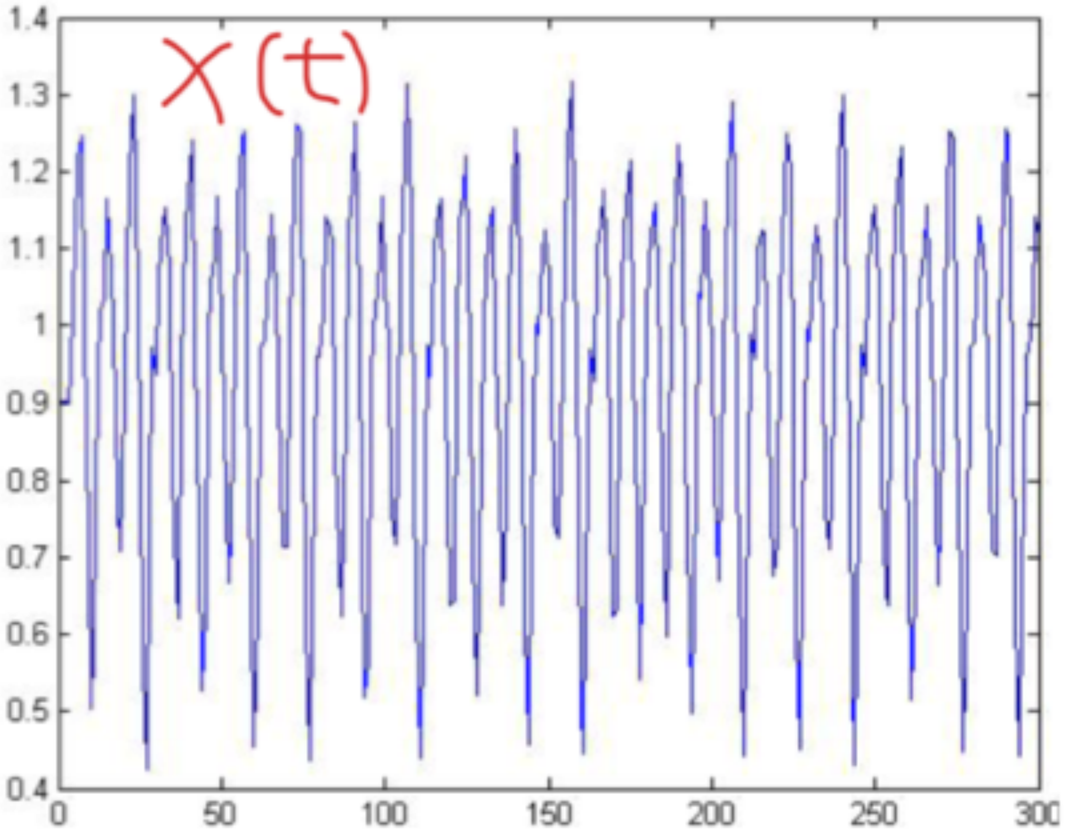
Runge-Kutta method for tracking differential equations

Chaotic differential equation

$$\frac{\partial x}{\partial t} = \frac{ax(t-\tau)}{1+x^{c}(t-\tau)} - bx(t)$$

a=0.2,c=10,b=0.1, tau=17

Given a differential equation, find x(t)



RK(Runge-Kutta)4

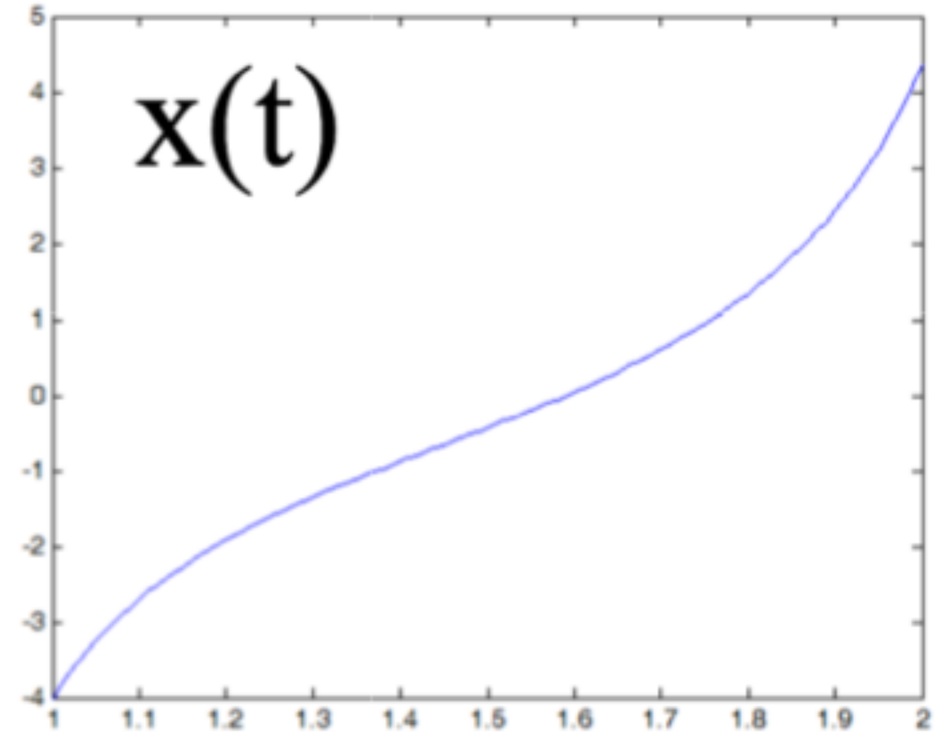
nonlinear delay differential equation

$$\frac{\partial x}{\partial t} = x(t-\tau) - x^3(1-\tau),$$

where the delay τ is set to 1.6.

$$x(0) = 0.2$$

Problem 1. State the formula of the Runge-Kutta 4 method for tracking differential equations. Draw a flow chart to illustrate solving the initial value problem by Runge-Kutta4 function (Numeric table)



x(b)

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ans =

   4.3712
```

4.371220807

a

b

Runge-Kutta 4

```
a:1
x(a) :-4
b:2
h:0.01
```

Differential equation

$$f(x,t) \triangleq \frac{\partial x}{\partial t} = 1 + x^2 + t^3$$

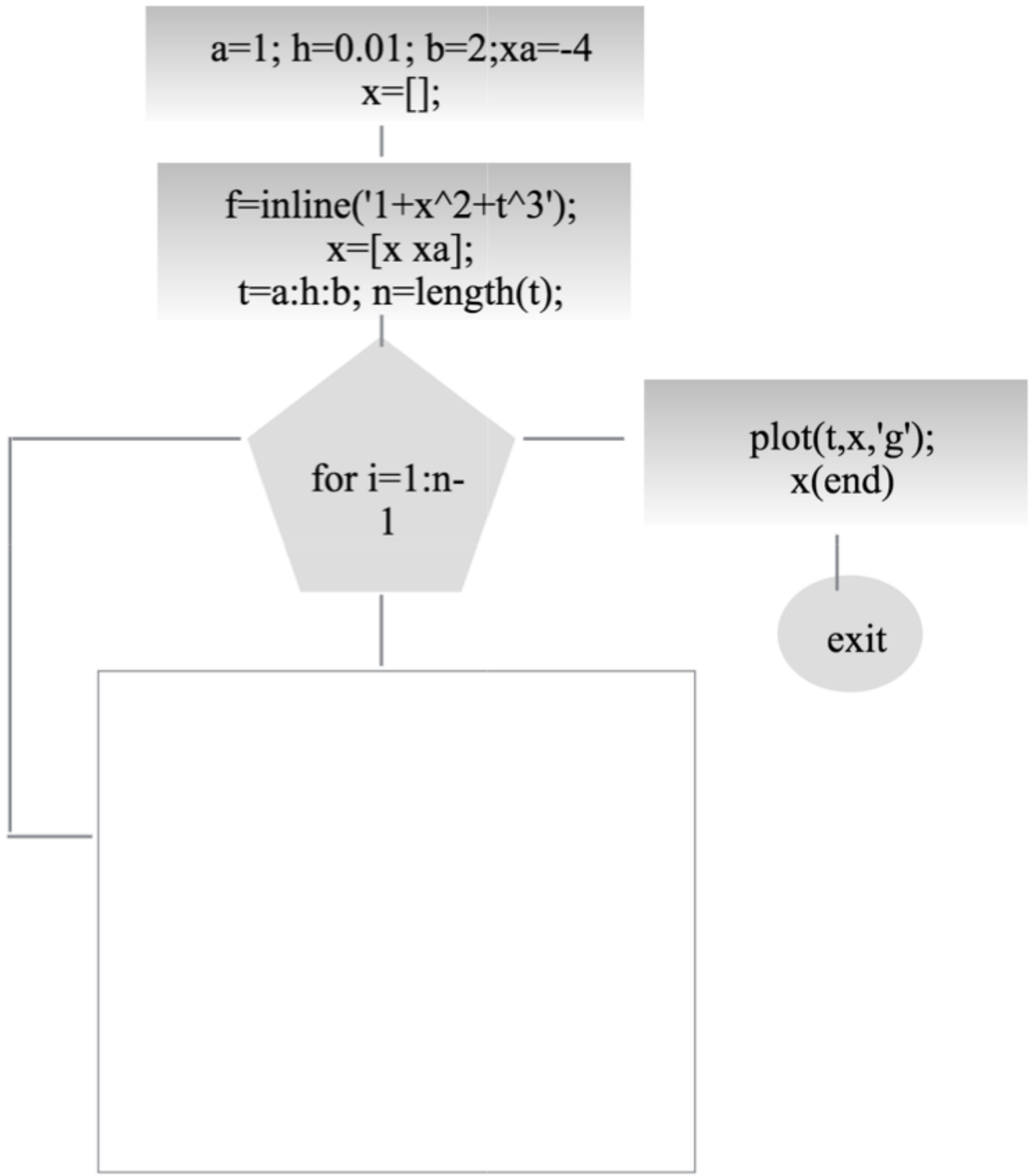
- Error for order m is $O\left(h^{m+1}\right)$ for each step of size h

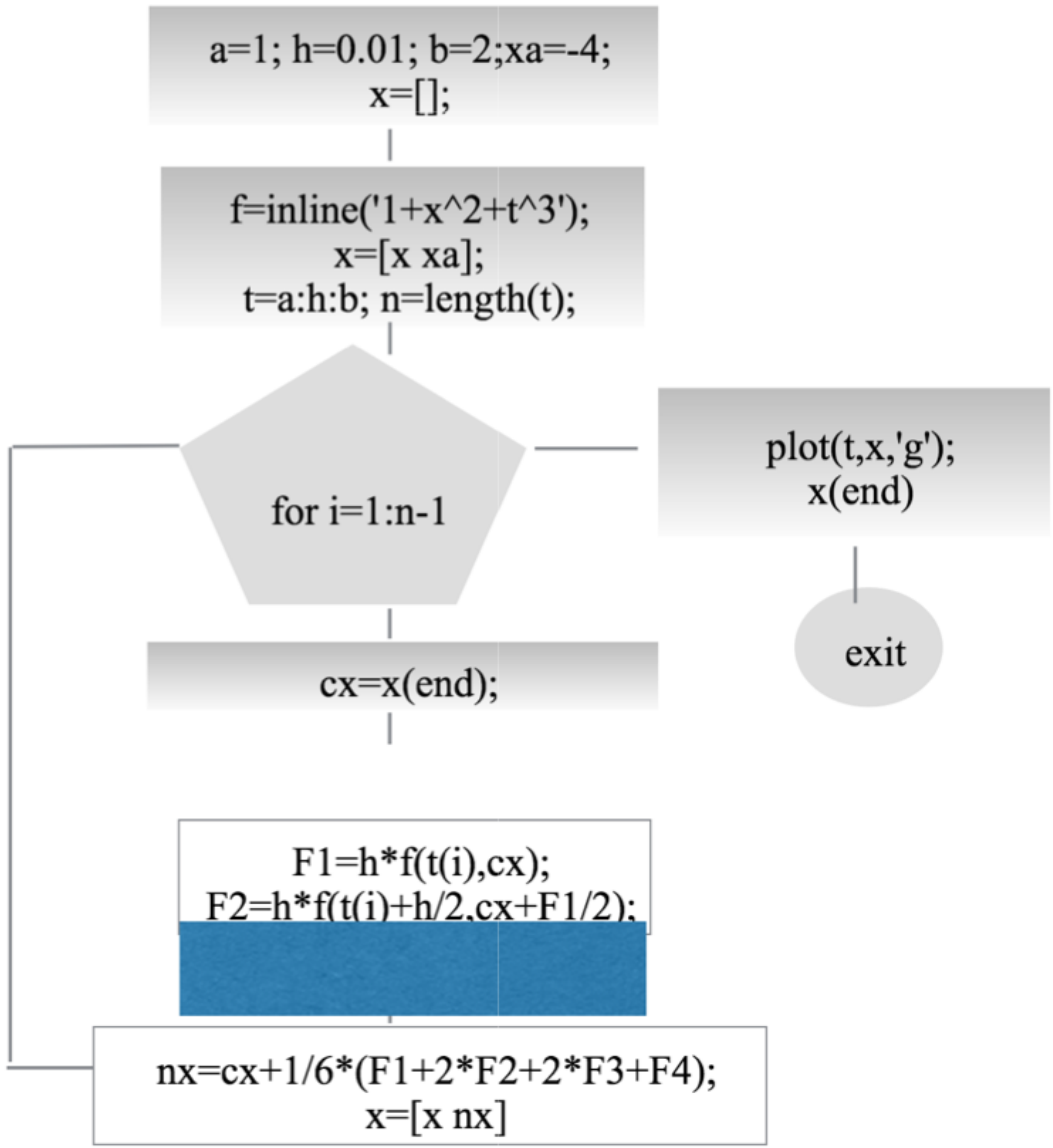
$h=0{,}01$

<u>RK4:</u>

$$x(t+h) = x(t) + \frac{1}{6}(F_1 + 2F_2 + 2F_3 + F_4)$$

$$\begin{cases} F_1 = hf(t,x) \\ F_2 = hf\left(t+\frac{1}{2}h, x+\frac{1}{2}F_1\right) \\ F_3 = hf\left(t+\frac{1}{2}h, x+\frac{1}{2}F_2\right) \\ F_4 = hf(t+h, x+F_3) \end{cases}$$


a=1; h=0.01; b=2;xa=-4
x=[];
f=inline('1+x^2+t^3');
x=[x xa];
t=a:h:b; n=length(t);
for i=1:n-1
plot(t,x,'g');
x(end)
exit


a=1; h=0.01; b=2;xa=-4;
x=[];
f=inline('1+x^2+t^3');
x=[x xa];
t=a:h:b; n=length(t);
for i=1:n-1
plot(t,x,'g');
x(end)
exit
cx=x(end);
F1=h*f(t(i),cx);
F2=h*f(t(i)+h/2,cx+F1/2);
[redacted]
nx=cx+1/6*(F1+2*F2+2*F3+F4);
x=[x nx]

Problem 2. Write matlab codes to implement your RK4 flow chart and test with the following IVP problem.

Runge-Kutta 4

a:1
x(a) :-4
b:2
h:0.01

Differential equation

$$f(x,t) \triangleq \frac{\partial x}{\partial t} = 1 + x^2 + t^3$$

```
for i=1:n-1
cx=x(end);
F1=h*f(t(i),cx);
F2=h*f(t(i)+h/2,cx+F1/2);

nx=cx+1/6*(F1+2*F2+2*F3+F4);
x=[x nx];
end
```

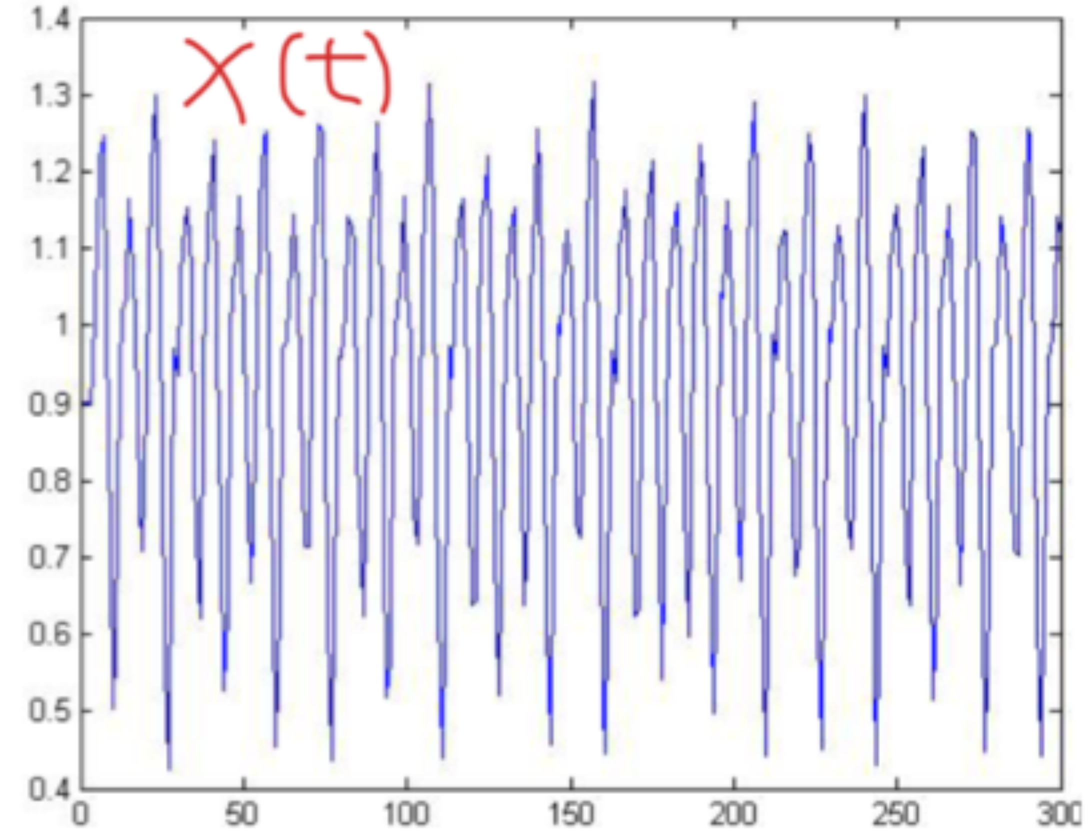
Problem 3. Apply matlab codes of RK4 to track the chaotic differential equation

Chaotic differential equation

$$\frac{\partial x}{\partial t} = \frac{ax(t-\tau)}{1+x^{c}(t-\tau)} - bx(t)$$

a=0.2,c=10,b=0.1, tau=17

Given a differential equation, find x(t)



Problem 4. Apply matlab codes of RK4 to track the nonlinear decay differential equation

RK(Runge-Kutta)4

nonlinear delay differential equation

$$\frac{\partial x}{\partial t} = x(t-\tau) - x^3(1-\tau),$$

where the delay τ is set to 1.6.

$$x(0) = 0.2$$