

1. (10 points) Give examples to explain matlab functions, poly and polyval.
2. Polynomial interpolation. Let  $x=[-1 \ 1 \ 3 \ 5]$ ,  $y=[105 \ -15 \ 9 \ -15]$ , and  $q$  denote a polynomial that well interpolates four points represented by  $x$  and  $y$ .
  - A. (5 points) State requirement of  $q$ .
  - B. (10 points) Let  $p_j$  be a polynomial, where  $j=1,\dots,4$ , satisfying
 
$$p_j(x(j)) = 1 \text{ and } p_j(x(i)) = 0 \text{ if } i \neq j,$$
    - i. Write codes to determine  $p_1$  that satisfies  $p_1(x(1))=1$  and  $p_1(x(i))=0$  for  $i=2,\dots,4$ . Note that  $p_1$  is termed as the first Lagrange polynomial determined by four elements of  $x$ .
    - ii. Write codes to determine  $q_1$  that  $q_1(x(1))=y(1)$  and  $q_1(x(i))=0$  for  $i=2,\dots,4$
  - C. (5 points) Let  $q=q_1 + q_2 + q_3 + q_4$ . Does  $q$  satisfy requirement in problem A? Why?
  - D. (5 points) Write codes to determine polynomial  $q$ .
3. (20 points) Let  $x=[x_1 \ x_2 \ \dots \ x_n]$  and  $y=[y_1 \ y_2 \ \dots \ y_n]$ .
  - A. Assume  $y_i = ax_i + b + e_i$ , where  $e_i$  denotes a random noise.
    - i. Let  $E(a,b)$  denote the mean square error of approximating  $y_i$  by  $ax_i + b$  for all  $i$ .  $E(a,b)=?$
    - ii. Derive the normal equation of minimizing  $E(a,b)$  with respect to  $a$  and  $b$ .
  - B. Assume  $y_i = ax_i^2 + bx_i + c + e_i$ .
    - i. Express the mean square error,  $E(a,b,c)$ , of approximating  $y_i$  by  $ax_i^2 + bx_i + c$  for all  $i$ .
    - ii. Derive the normal equation of minimizing  $E(a,b,c)$
4. (10 points) Draw a flow chart to illustrate minimizing  $E(a,b,c)$  with respect to  $a$ ,  $b$  and  $c$  for given  $x$  and  $y$
5. (10 points) Write a matlab function to implement the flow chart.
6. (25 points) Let  $a=3$ ,  $b=-2$  and  $c=1$ . Use the matlab function in 5 to find  $a$ ,  $b$  and  $c$  and plot the following figure. Checked by \_\_\_\_\_ time:

