Reviews

RK4:

h=

$$x(t+h) = x(t) + \frac{1}{6}(F_1 + 2F_2 + 2F_3 + F_4)$$

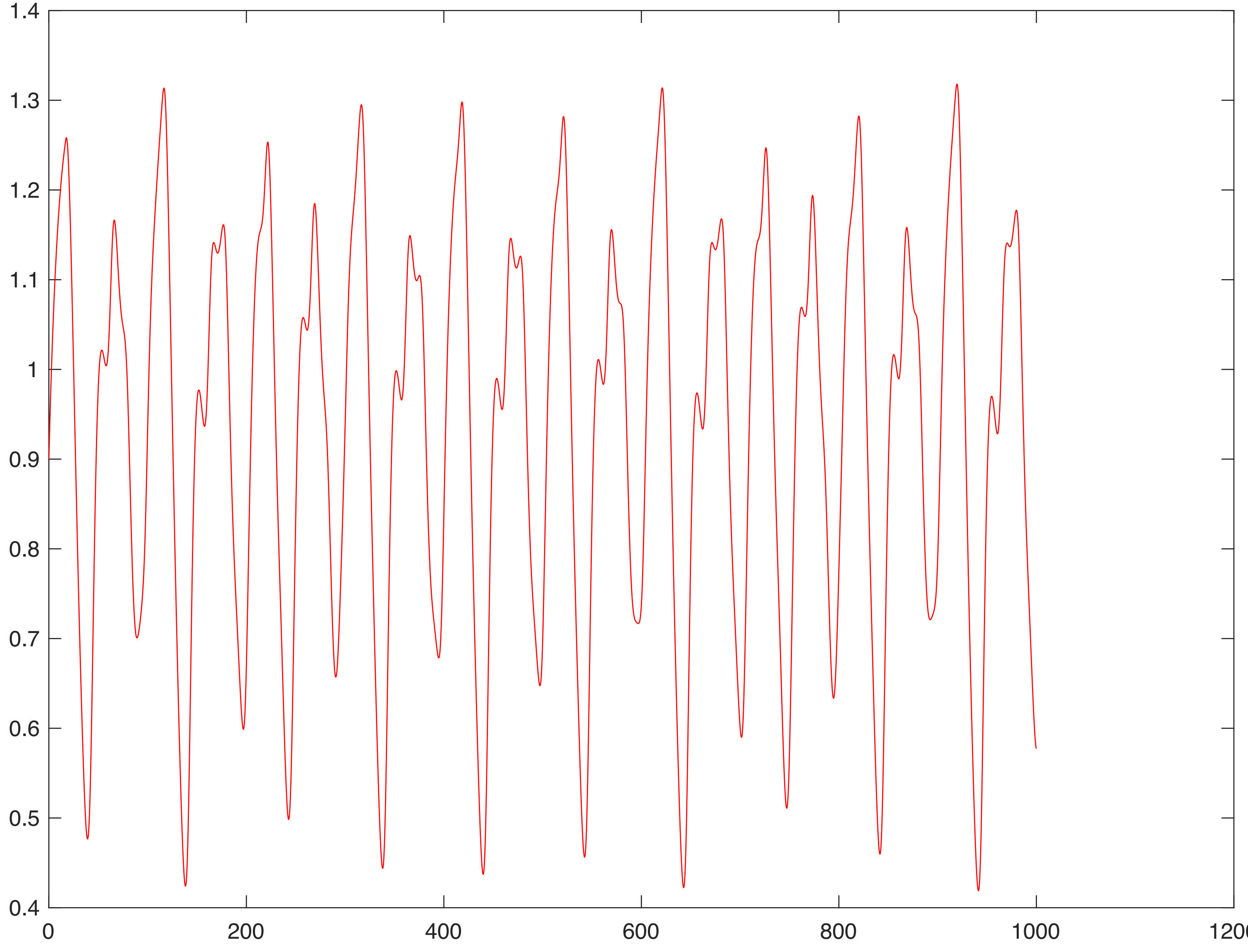
$$\begin{cases} F_1 &= hf(t, x) \\ F_2 &= hf\left(t + \frac{1}{2}h, x + \frac{1}{2}F_1\right) \\ F_3 &= hf\left(t + \frac{1}{2}h, x + \frac{1}{2}F_2\right) \\ F_4 &= hf(t + h, x + F_3) \end{cases}$$

My RK4 for MG17

```
function demo_RK4_MG()
data=[0;0.9];  % row 1 stores t values, row 2 storing x values
h=0.1;
t=0;
for i=1:1000/h
    L=size(data,2); cx=data(2,L);
    F1=h*mg(data,t,cx);
    F2=h*mg(data,t+h/2,cx+F1/2);
    F3=h*mg(data,t+h/2,cx+F2/2);
    F4=h*mg(data,t+h,cx+F3);
    nx=cx+1/6*(F1+2*F2+2*F3+F4);
    t=t+h;
    data=[data [t nx]'];
end
plot(data(1,:),data(2,:),'g');
return
```

```
function ans=mg(data,t,x)
if t-17 < 0
   px = data(2,1);
else
   d=abs(data(1,:)-(t-17));
   [v ind]=min(d);
   px=data(2,ind);
end
ans=0.2*px/(1+px^10)-0.1*x;
```

Find the previous t and set its x value to px


1.4
1.3
1.2
1.1
1
0.9
0.8
0.7
0.6
0.5
0.4
0
200
400
600
800
1000
1200

Derivative depends on current and previous x values

Chaotic differential equation

$$\frac{\partial x}{\partial t} = \frac{a x(t-\tau)}{1 + x^{c}(t-\tau)} - b x(t)$$

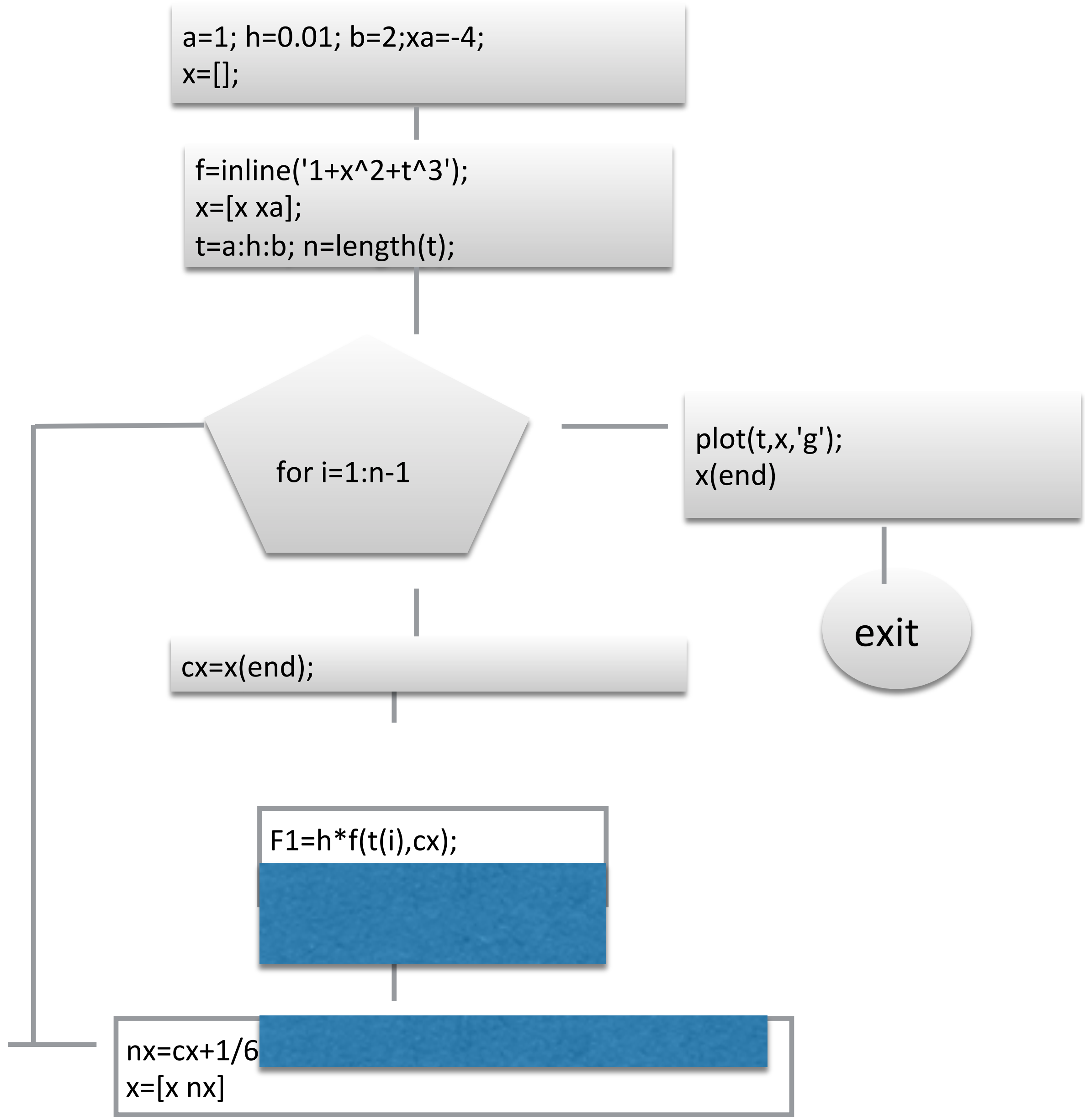
a=0.2,c=10,b=0.1, tau=17

t-x table

t	0.0	0.1	0.2	0.3	0.4	0.5			
x	0.9000	0.9043	0.9086	0.9128	0.9170	…			

Derivative depends on current x

$$f(x,t) \triangleq \frac{\partial x}{\partial t} = 1 + x^2 + t^3$$



a=1; h=0.01; b=2;xa=-4;
x=[];
f=inline('1+x^2+t^3');
x=[x xa];
t=a:h:b; n=length(t);
for i=1:n-1
plot(t,x,'g');
x(end)
exit
cx=x(end);
F1=h*f(t(i),cx);
nx=cx+1/6
x=[x nx]

Derivation of formula

$$f'(x) = \frac{f(x+h) - f(x-h)}{2h} - \frac{1}{6}h^2 f'''(x)$$

$$f(x \pm h) =$$

$$f(x) \pm h f'(x) + h^2 \frac{f''(x)}{2!} \pm h^3 \frac{f'''(x)}{3!} + h^4 \frac{f^{(4)}(x)}{4!} \pm h^5 \frac{f^{(5)}(x)}{5!} + \cdots$$

$$f(x+h) - f(x-h) = 2hf'(x) + 2h^3 \frac{f'''(x)}{3!} + 2h^5 \frac{f^{(5)}(x)}{5!} + \cdots$$

Derivation of Richardson Extrapolation

Halving the stepsize, $\therefore$

$$\begin{aligned}
\phi(h) &= f'(x) - a_2h^2 - a_4h^4 - a_6h^6 - \cdots \\
\phi\left(\frac{h}{2}\right) &= f'(x) - a_2\left(\frac{h}{2}\right)^2 - a_4\left(\frac{h}{2}\right)^4 - a_6\left(\frac{h}{2}\right)^6 - \cdots \\
\phi(h) - 4\phi\left(\frac{h}{2}\right) &= -3f'(x) - \frac{3}{4}a_4h^4 - \frac{15}{16}a_6h^6 - \cdots
\end{aligned}$$

- Divide by 3 and write $f'(x)$

$$\begin{aligned}
f'(x) &= \frac{4}{3}\phi\left(\frac{h}{2}\right) - \frac{1}{3}\phi(h) - \frac{1}{4}a_4h^4 - \frac{5}{16}a_6h^6 - \cdots \\
&= \phi\left(\frac{h}{2}\right) + \underbrace{\frac{1}{3}\left[\phi\left(\frac{h}{2}\right) - \phi(h)\right]}_{\equiv(*)} + O\left(h^4\right)
\end{aligned}$$

Problem 2. Apply jacobian.m to find the Jacobian of the following coordinate functions

A. Derive Jacobian

B. Write matlab codes to obtain an inline function for calculation of Jacobian

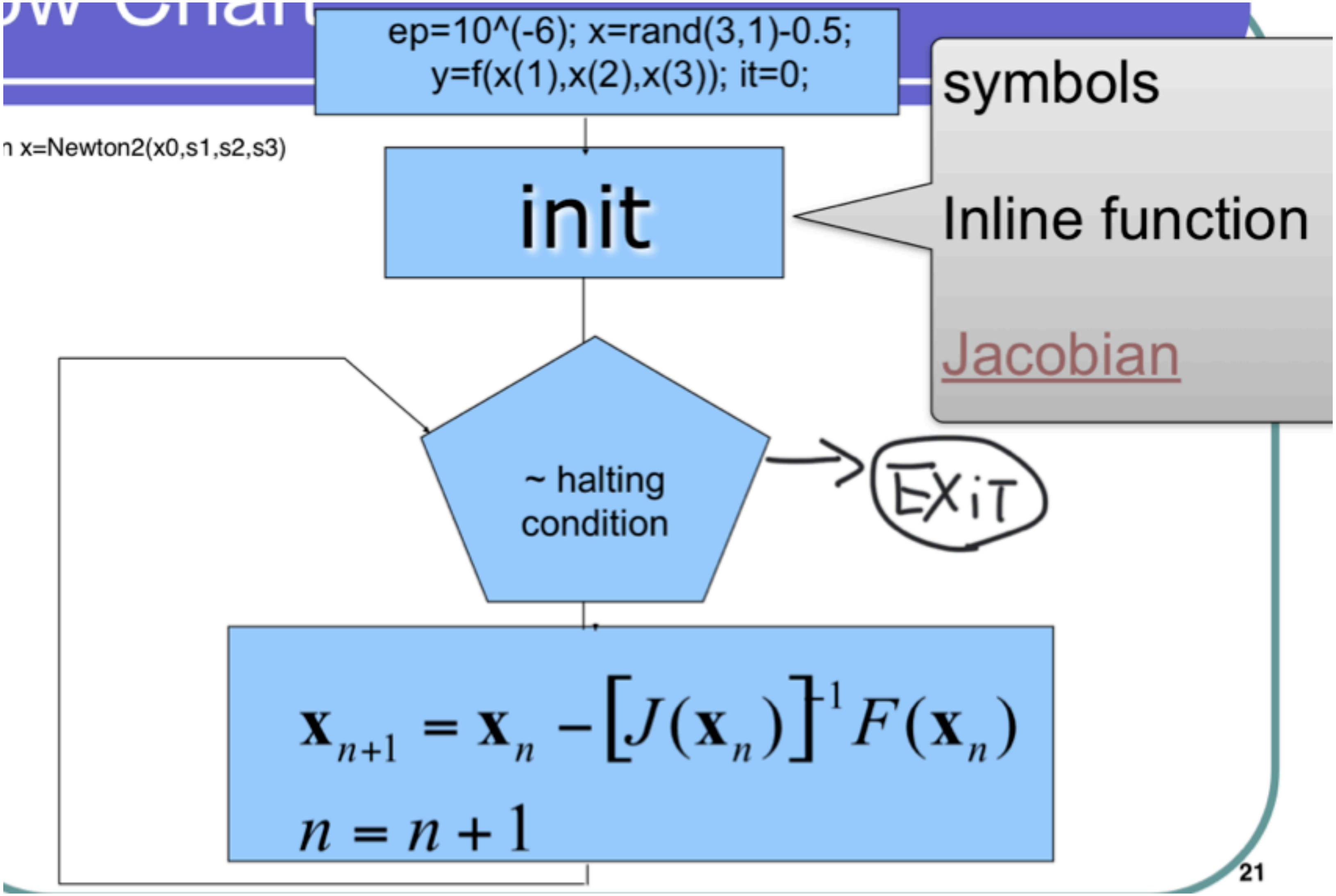
$$f_1(x_1, x_2) = x_1^2 + x_2^2 - 1$$

$$f_2(x_1, x_2) = x_1^2 - x_2^2$$

Updating rule

$$\mathbf{x}_{n+1} = \mathbf{x}_n - \left[J(\mathbf{x}_n)\right]^{-1} F(\mathbf{x}_n)$$

Flow chart



ep=10^(-6); x=rand(3,1)-0.5;
y=f(x(1),x(2),x(3)); it=0;
x=Newton2(x0,s1,s2,s3)
init
symbols
Inline function
Jacobian
~ halting condition
EXIT
$\mathbf{x}_{n+1} = \mathbf{x}_n - [J(\mathbf{x}_n)]^{-1} F(\mathbf{x}_n)$
$n = n + 1$
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Use the method of Gauss-Jordan elimination to find reduced echelon form of the following matrix.

$$\begin{bmatrix} 0 & 0 & 2 & -2 & 2 \\ 3 & 3 & -3 & 9 & 12 \\ 4 & 4 & -2 & 11 & 12 \end{bmatrix}$$

Solution

Backward substitution

$$\left(\begin{array}{cccc|c} 6 & -2 & 2 & 4 & 16 \\ 0 & -4 & 2 & 2 & -6 \\ 0 & 0 & 2 & -5 & -9 \\ 0 & 0 & 0 & -3 & -3 \end{array}\right)$$

$$6x_1 - 2\times 1 - 2\times 2 + 4\times 1 = 16 \Rightarrow x_1 = 3$$

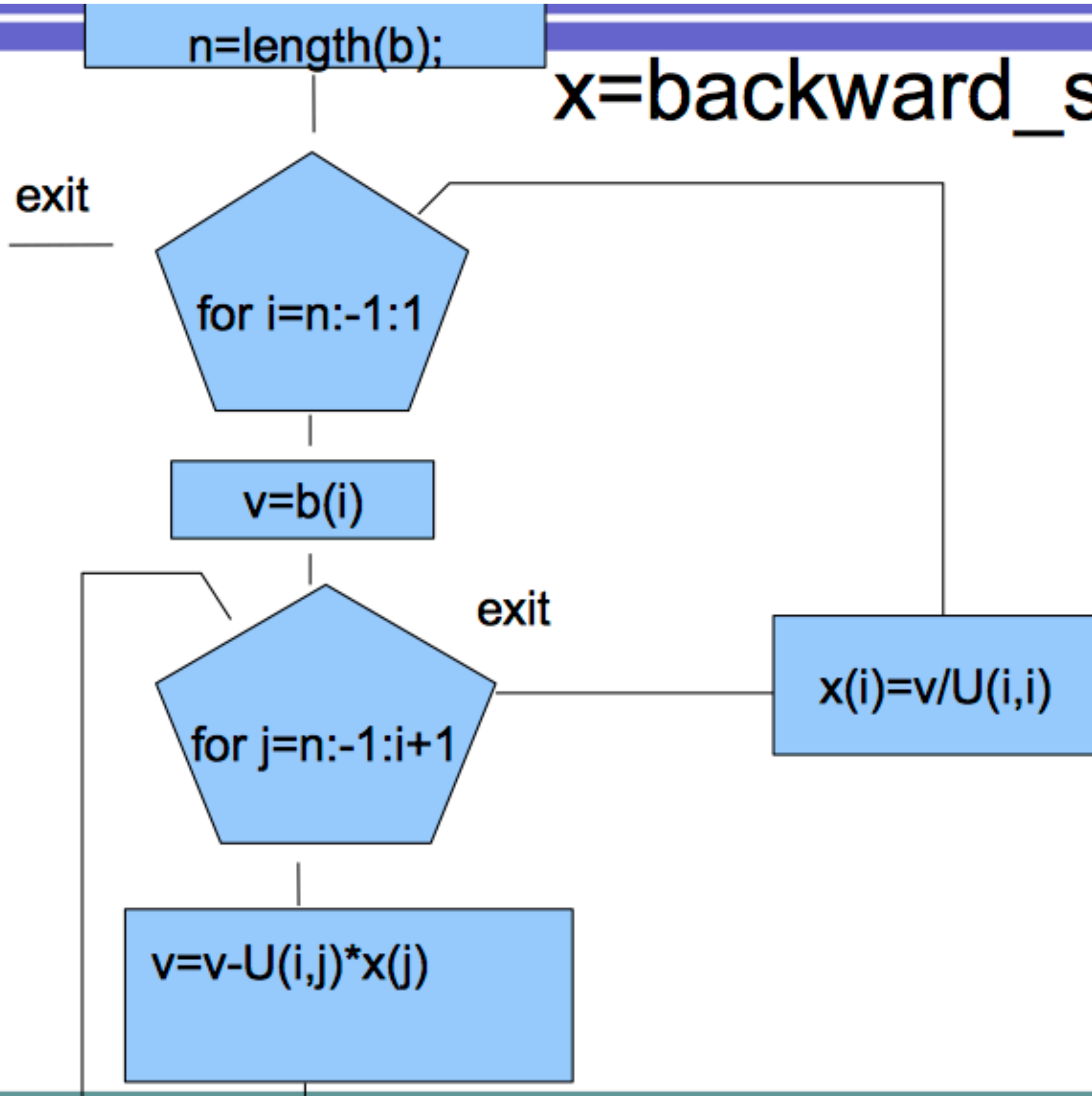
$$-4x_2 - 2\times 2 + 2\times 1 = -6 \Rightarrow x_2 = 1$$

$$2x_3 - 5\times 1 = -9 \Rightarrow x_3 = -2$$

$$x_4 = 1$$

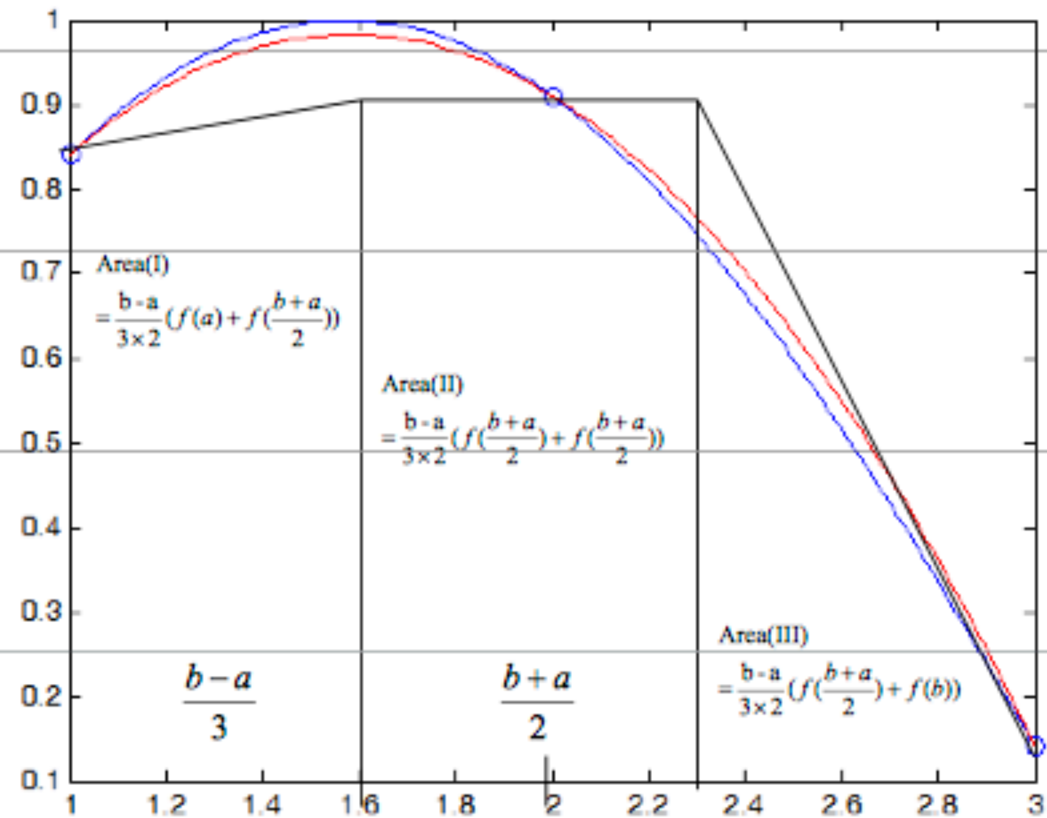
U(i,i) is not zero for all i

x=backward_sub(U,b)



$$\text{Area(I)} + \text{Area(II)} + \text{Area(III)}$$

$$= \frac{\text{b - a}}{3 \times 2}\left(f(a) + 4f\left(\frac{b+a}{2}\right) + f(b)\right)$$



Composite Simpson rule

$$\int_a^b f(x)dx = \sum_{i=0}^{n-1} \int_{a+2ih}^{a+(2i+2)h} f(x)dx$$

$$\approx \frac{h}{3} \sum_{i=0}^{n-1} (f(a+2ih) + 4f(a+(2i+1)h) + f(a+(2i+2)h))$$

$$\int_a^b f(x)dx = \sum_{i=0}^{n-1} \int_{a+2ih}^{a+(2i+2)h} f(x)dx$$

$$\approx \frac{h}{3} \sum_{i=0}^{n-1} (f(a+2ih) + 4f(a+(2i+1)h) + f(a+(2i+2)h))$$