1. Refer to the following code

```
function a=gradient descent(x,y)
max loop=2000;
[\mathbf{M} \cdot \mathbf{d}]=size(x);
X=[x \text{ ones}(N,1)];a=rand(1,d+1); hc=0; c=0.1;e=y-X*a';
E=100; loop=1;
while \neghc
  G=mean(X.*(e*ones(1,d+1)));
  a new=a-c*G;
  y hat=X*a new';
  e new=y hat-y;
  E new=mean(e new.^2);
  if mod(loop, 100)==0
  fprintf('loop %d mse %f\n',loop,E_new);
  end
  if E_{new} < E & loop \leq max_loop
     a=a_new;e=e_new;
     E=E new;
  else
     hc=1;
  end
  loop = loop + 1;
end
```
- A. Figure out the architecture of a linear relation.
- B. Write codes for error calculation.
- C. Write codes for gradient calculation.
- D. Write codes for y\_hat calculation.
- 2. Consider the following linear relations



A. Let a<sub>1</sub> and a<sub>2</sub> be columns of matrix a. Let  $(x_i \; b_{ii})$  be paired data, where i runs from 1 to n.

$$
E(\mathbf{a}) = \frac{1}{2n} \sum_{i=1}^{n} \sum_{j=1}^{2} (\mathbf{x}_i^T \mathbf{a}_j - b_{ji})^2
$$

Derive 
$$
\frac{d}{d\mathbf{a}}E(\mathbf{a})
$$

- B. Write codes for error calculation.
- C. Write codes for gradient calculation.
- D. Write codes for y\_hat calculation.
- 3. Consider the following nonlinear relations



A. Let

$$
E(a) = \frac{1}{2n} \sum_{i=1}^{n} \sum_{j=1}^{2} (\tanh(x_i^T a_j) - b_{ji})^2
$$

Derive *d*  $\frac{d}{dx}E(\mathbf{a})$ 

- B. Write codes for error calculation
- C. Write codes for gradient calculation.
- D. Write codes for y\_hat calculation.
- 4. Write function gradient\_descent to estimate the matrix for linear transformation

$$
x = rand(400,2);
$$
  
\n $z(:,1) = 2*x(:,1)+x(:,2)-1;$   
\n $z(:,2)=x(:,1)-x(:,2)+1;$   
\n $a = gradient\_descent(x,z)$   
\nChecked by \_\_\_\_\_\_\_\_time \_\_\_\_\_\_\_\_

5. Write function gradient\_descent to estimate the matrix for nonlinear transformation

x=rand(400,2);

 $z(:,1) = \tanh(2*x(:,1) + x(:,2) - 1);$ 

 $z(:,2) = \tanh(x(:,1)-x(:,2)+1);$ 

a=gradient\_descent(x,z)