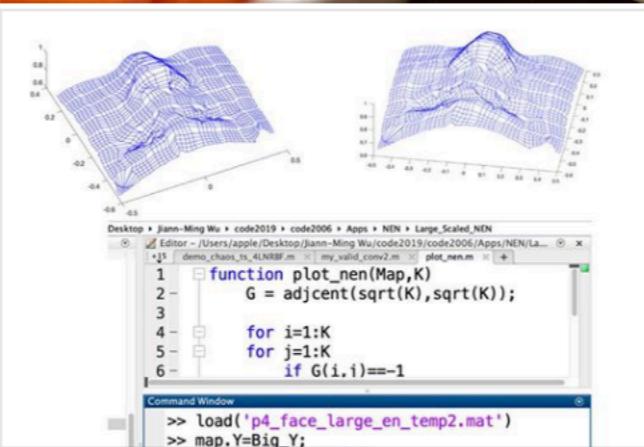


# Matlab

# Taylor expansion

$$f(a + \delta) \sim f(a) + \sum_{k=1}^K \frac{1}{k!} f^k(a) \delta^k$$

## 人工智慧基礎課程 備課點心



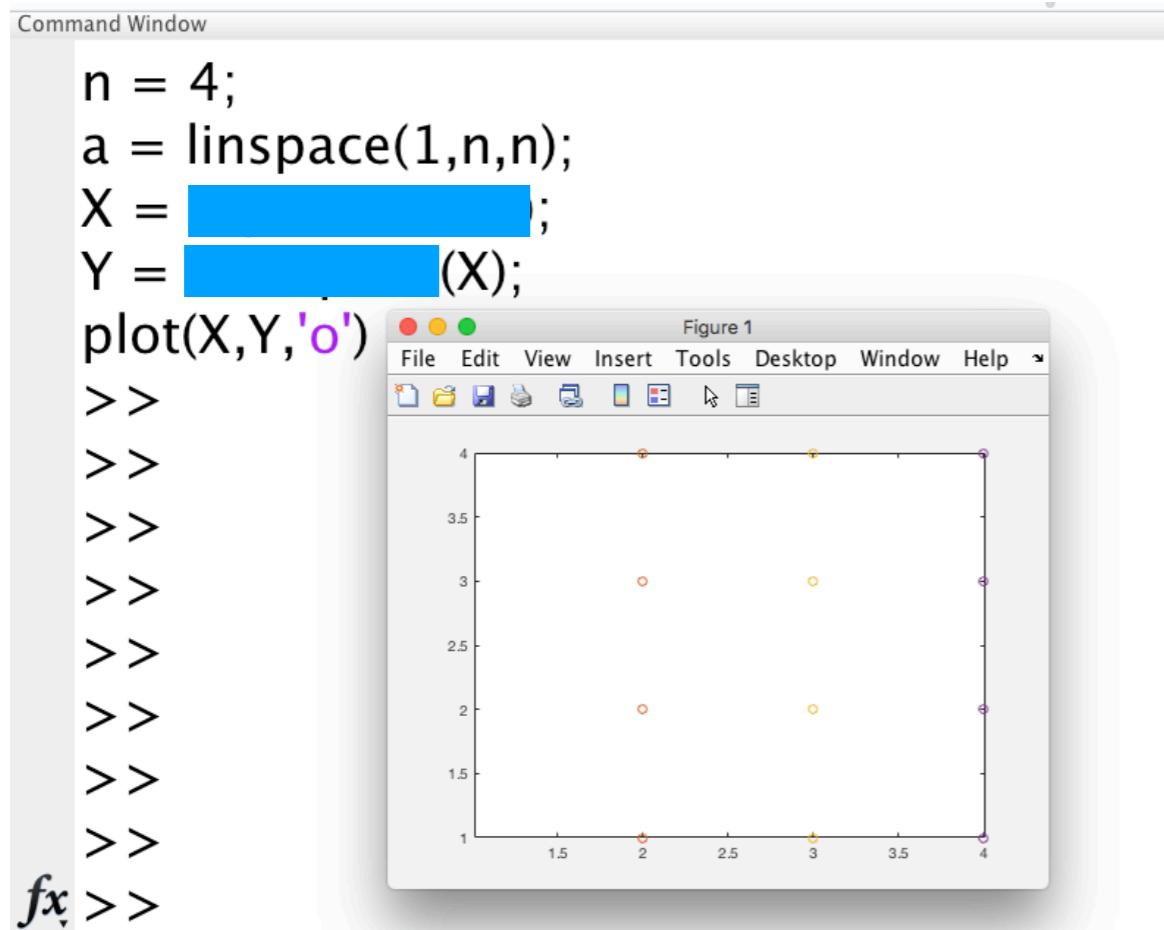
## Exercise

```
function demo_ex()
% step 1. Apply Matlab statement taylor to
%         find Maclaurin series expansion of exp(-x^2)
%         and set it to p

% step 2. Translate p to an inline px
% step 3. Set a = 1/2 and delta = 0.01
%         and substitute a + delta to inline px
```

右圖中十六個點呈 $4 \times 4$ 方陣排列，矩陣X的元素代表相對應方陣點的橫坐標，Y矩陣的元素代表相對應方陣點的縱坐標。請完成下列程式，畫出方陣點

```
Command Window
n = 4;
a = linspace(1,n,n);
X = [ ] ;
Y = [ ] (X);
plot(X,Y, 'o')
>>
>>
>>
>>
>>
>>
>>
>>
fx >>
```



The screenshot shows the MATLAB Command Window with the following code:

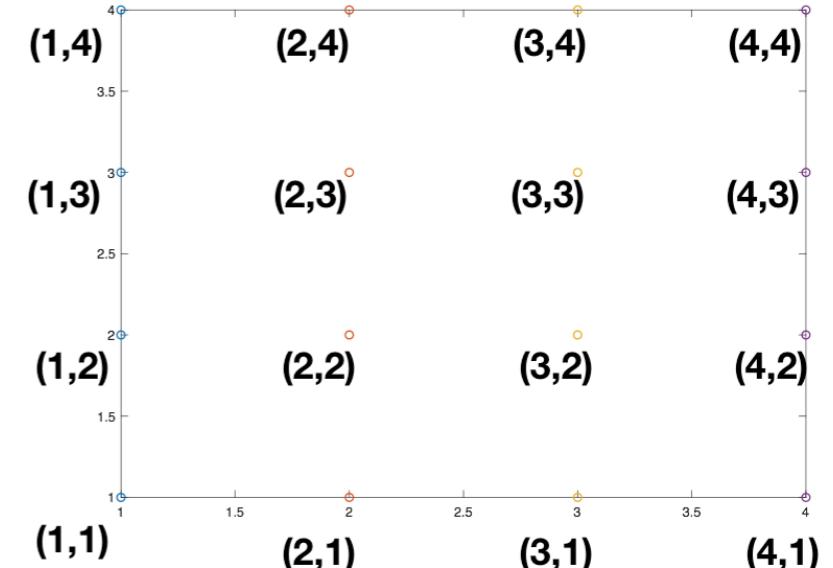
```
n = 4;
a = linspace(1,n,n);
X = [ ] ;
Y = [ ] (X);
plot(X,Y, 'o')
```

After running the code, a plot window titled "Figure 1" is displayed, showing a 4x4 grid of points at integer coordinates from (1,1) to (4,4). The plot has x and y axes ranging from 1 to 4.

$$a = [1 2 3 4]$$

$$X = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{bmatrix}$$

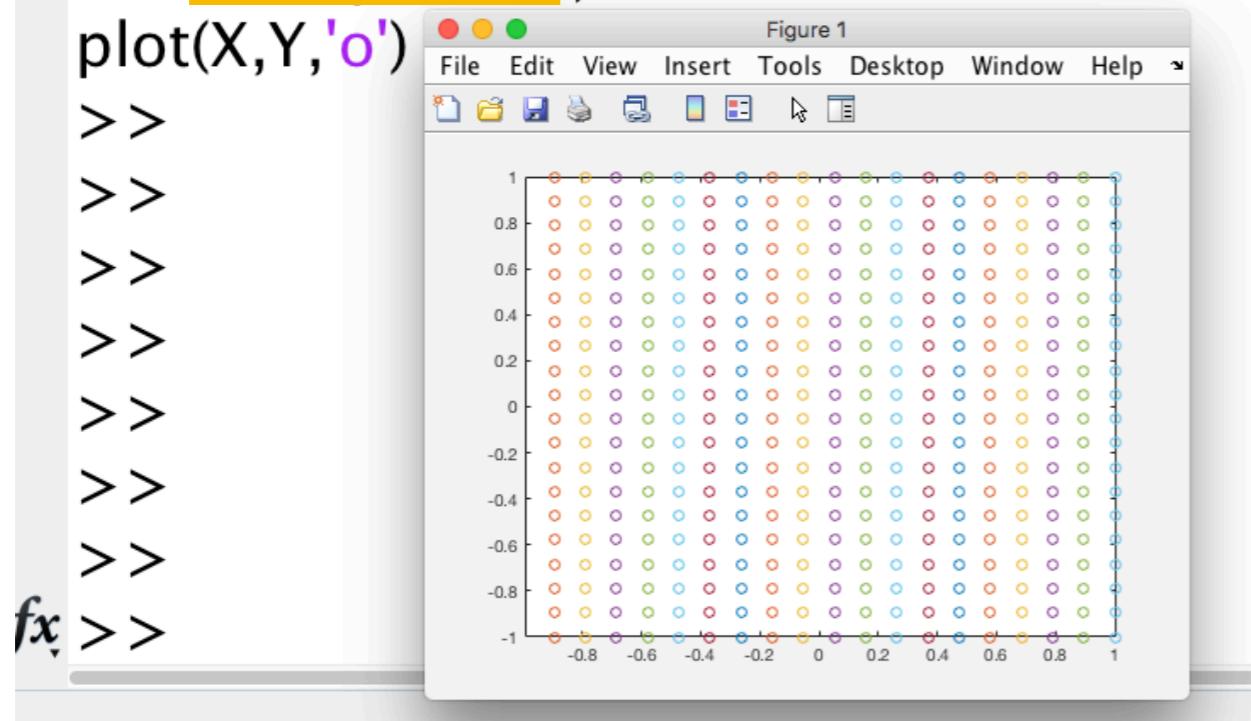
$$Y = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 \\ 3 & 3 & 3 & 3 \\ 4 & 4 & 4 & 4 \end{bmatrix}$$



在 $[-1, 1] \times [-1, 1]$ 的區域中繪製 $20 \times 20$ 的點矩陣

```
n = 20;  
a = [ ];  
X = [ ];  
Y = [ ];  
plot(X,Y,'o')
```

本題答題

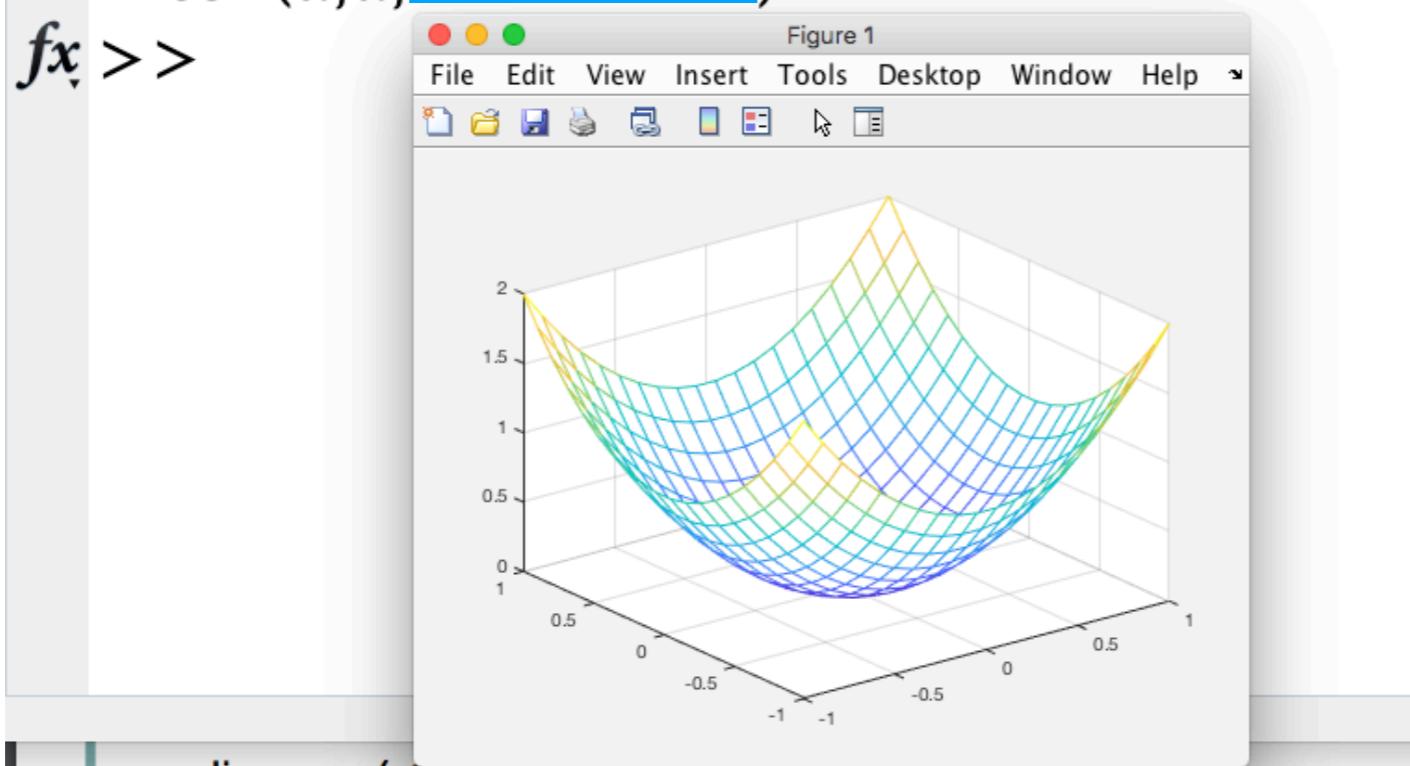


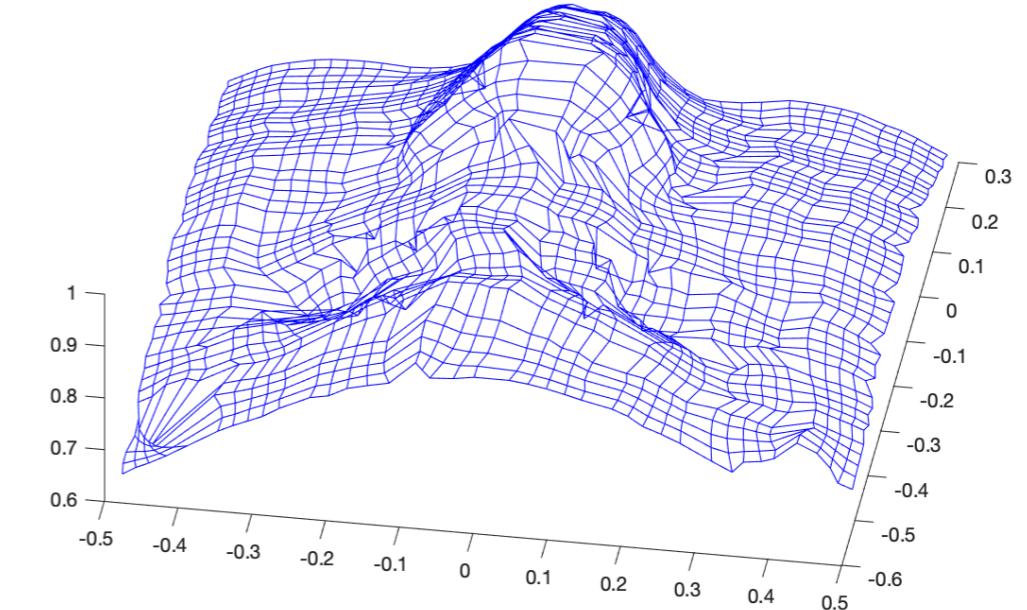
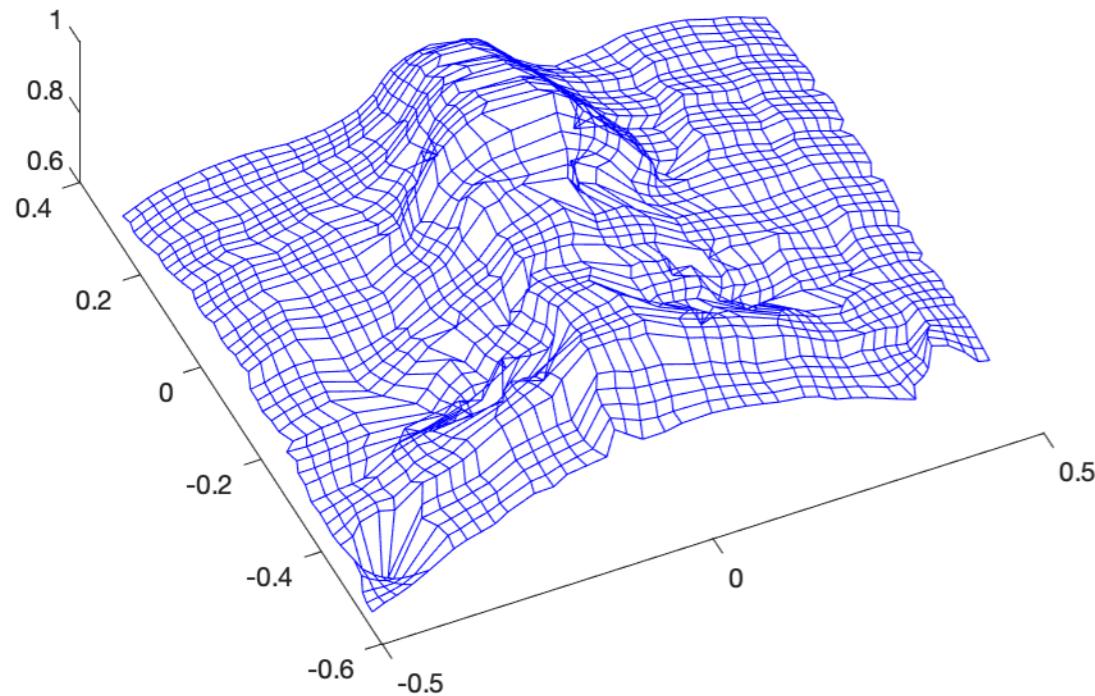
請完成mesh指令，繪製  $f(x, y) = x^2 + y^2$  的立體圖

```
n = 20;  
a = [ -1 : 1 / n : 1 ];  
X = [ -1 : 1 / n : 1 ];  
Y = [ -1 : 1 / n : 1 ];  
mesh(a,a, )
```

*fx >>*

本題答題





Desktop ▶ Jiann-Ming Wu ▶ code2019 ▶ code2006 ▶ Apps ▶ NEN ▶ Large\_Scaled\_NEN

Editor - /Users/apple/Desktop/Jiann-Ming Wu/code2019/code2006/Apps/NEN/La... x

+15 demo\_chaos\_ts\_4LNRBF.m x my\_valid\_conv2.m x plot\_nen.m x +

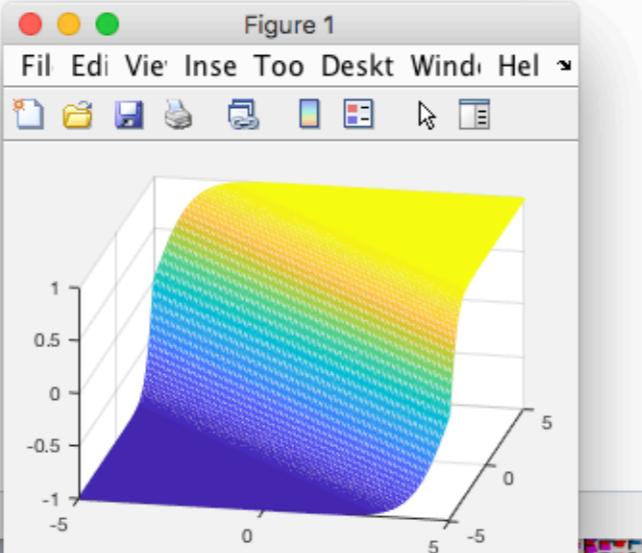
```
1 function plot_nen(Map,K)
2 G = adjcent(sqrt(K),sqrt(K));
3
4 for i=1:K
5 for j=1:K
6 if G(i,j)==-1
```

Command Window

```
>> load('p4_face_large_en_temp2.mat')
>> map.Y=Big_Y;
>> plot_nen(map,40*40)
>> plot_nen(map,40*40)
```

請完成mesh指令，繪製  $f(x, y) = \tanh(x + y)$  的立體圖

```
n = 100;
a = linspace(-5,5,n);
X = [REDACTED];
Y = [REDACTED];
mesh(a,a,[REDACTED])
>> <span style="color: red;">f(x) >>
```

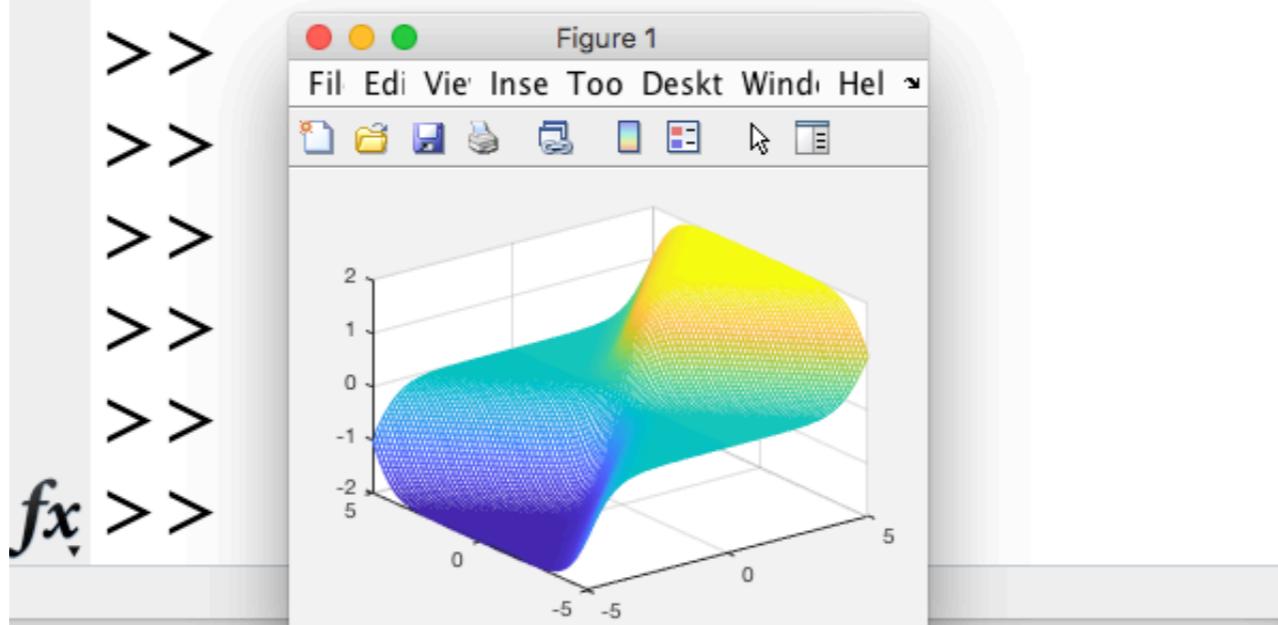
A screenshot of a MATLAB command window titled "Figure 1". The window shows a 3D surface plot of the function f(x,y) = tanh(x+y). The surface is a hyperbolic paraboloid, resembling a saddle shape, colored with a gradient from blue at the bottom to yellow at the top. The x and y axes range from -5 to 5, and the z-axis ranges from -1 to 1. The plot is displayed on a wireframe grid.

本題答題

請完成mesh指令，繪製  $f(x, y) = \tanh(x + y) + \tanh(x - y)$  的立體圖

```
n = 100;  
a = linspace(-5,5,n);  
X = [REDACTED];  
Y = [REDACTED];  
mesh(a,a,[REDACTED])
```

本題答題



Find the Maclaurin series expansions of the exponential, sine, and cosine functions up to the fifth order.

```
syms x
T1 = taylor(exp(x))
T2 = taylor(sin(x))
T3 = taylor(cos(x))
```

$$f(a + \delta) \sim f(a) + \sum_{k=1}^K \frac{1}{k!} f^k(a) \delta^k$$

# Exercise taylor\_ex1

```
function taylor_ex1()
% step 1. Apply Matlab statement taylor to
%           find Maclaurin series expansion of exp(-x^2)
%           and set it to p

% step 2. Translate p to an inline px

% step 3. Set a = 1/2 and delta = 0.01
%           and substitute a + delta to inline px
```

PLOTS APPS EDITOR PUBLISH FILE VERSIONS VIEW

**B** Bold **I** Italic **Σ** Inline LaTeX **M** Monospaced

ON INSERT INLINE MARKUP

**B** Bulleted List **I** Preformatted Text  
**N** Numbered List **C** Code  
**Image** **D** Display LaTeX

INSERT BLOCK MARKUP

**P** Publish as HTML **P** Publish as PDF

PUBLISH

/ > MATLAB Drive >

third\_derivatives.m demo\_Taylor\_ex1\_jmwu.m Figure 1 +

s.m

```
function demo_Taylor_ex1_jmwu()
    syms x;
    p = taylor(exp(x));
    px = inline(p);
    a = linspace(-5,5,100);
    plot(a,px(a))
end
```

New Script

Figure 1: A plot of the Taylor series approximation of the exponential function  $e^x$ . The x-axis ranges from -5 to 5, and the y-axis ranges from -20 to 100. The curve starts at approximately (-5, -17) and ends at approximately (5, 140), passing through the origin (0, 0).

SIZE CLASS

.. 1x55 char

COMMAND WINDOW

New to MATLAB? See resources for [Getting Started](#).

matlab.mathworks.com

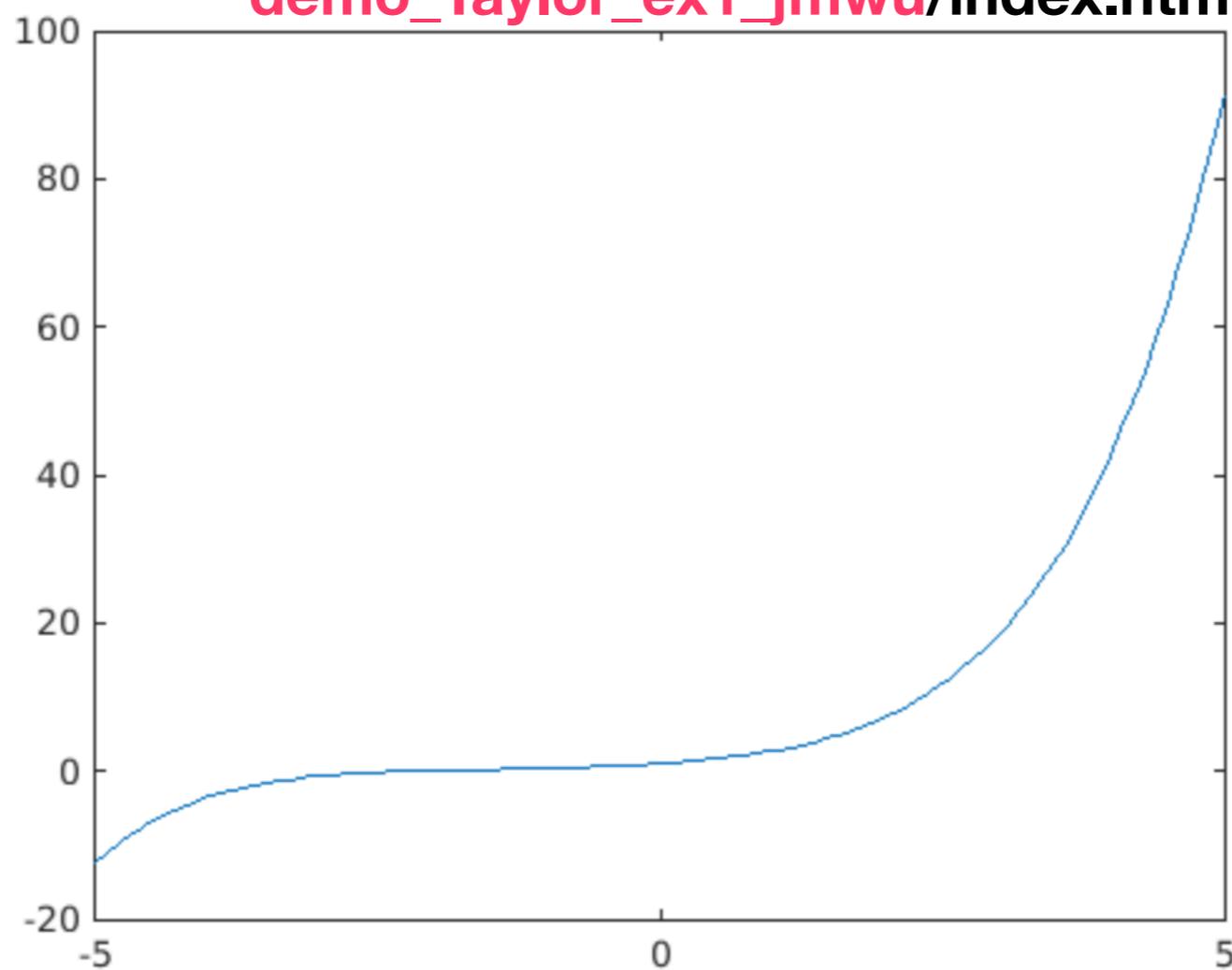
MATLAB Online R2019b

```
function demo_Taylor_ex1_jmwu()

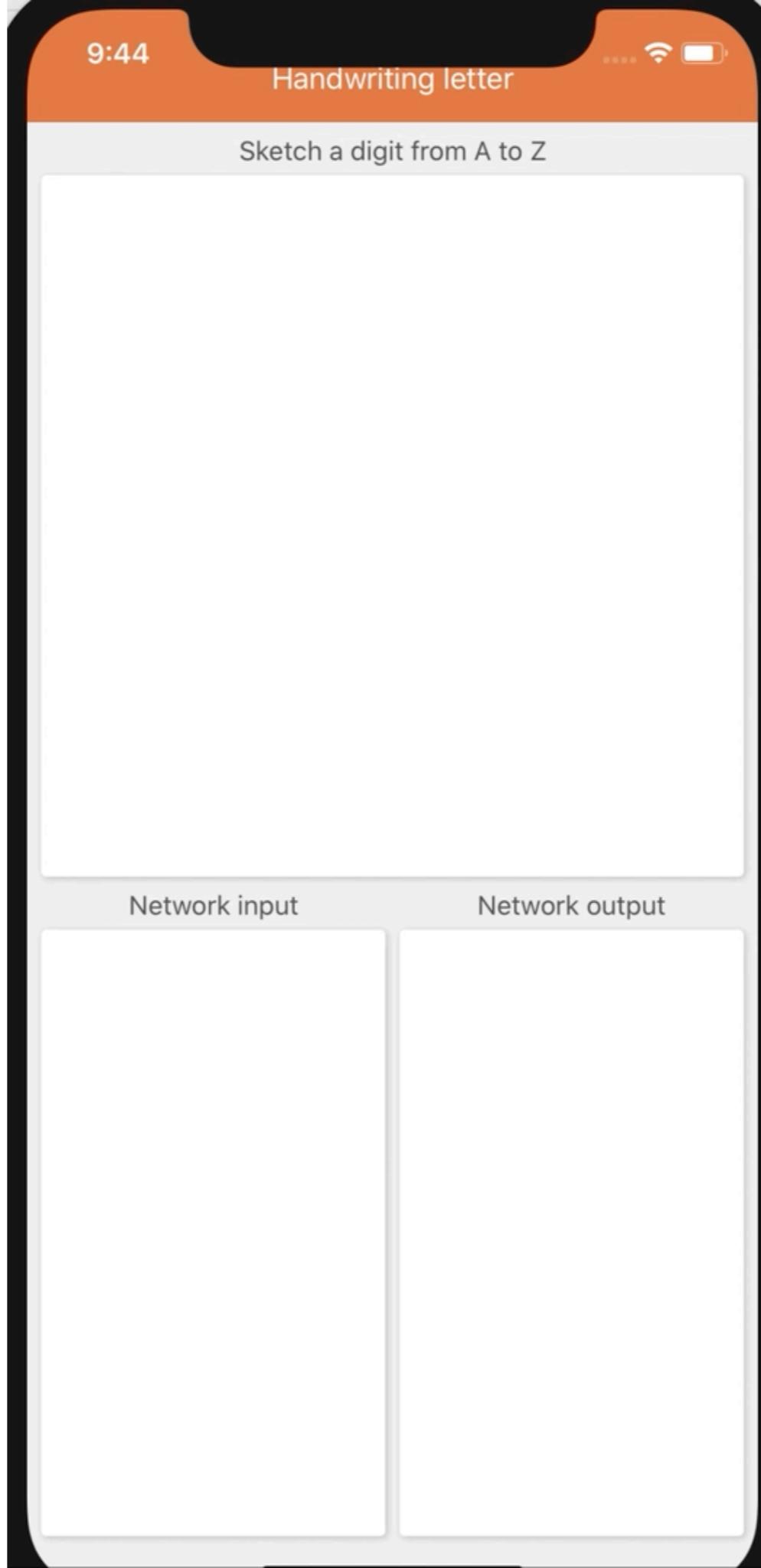
syms x;
p = taylor(exp(x));
px = inline(p);
a = linspace(-5,5,100);
plot(a,px(a))

end
```

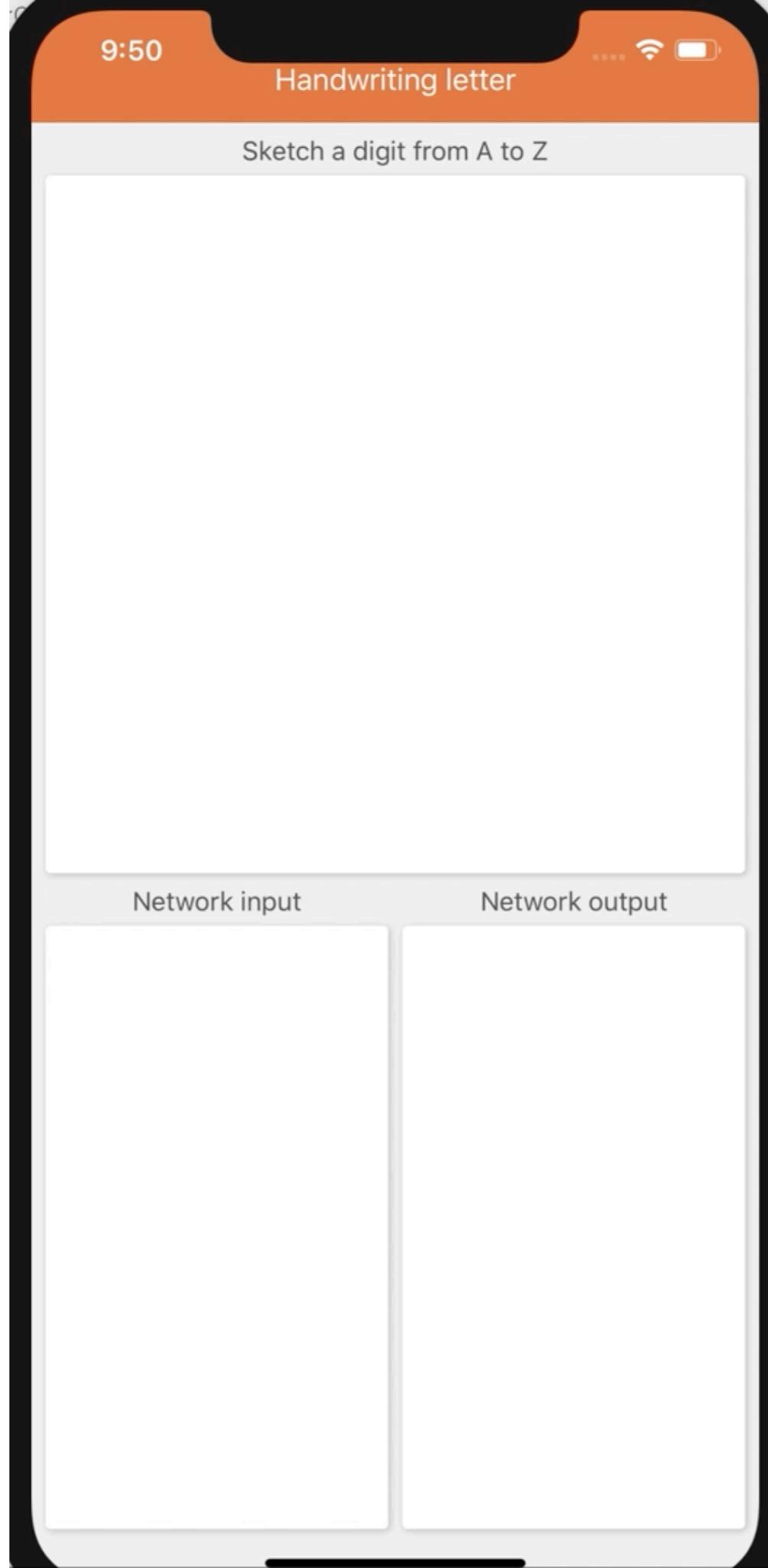
[https://matlab.mathworks.com/users/jmwu@mail.ndhu.edu.tw/Published/  
demo\\_Taylor\\_ex1\\_jmwu/index.html](https://matlab.mathworks.com/users/jmwu@mail.ndhu.edu.tw/Published/demo_Taylor_ex1_jmwu/index.html)



**LetterCore36**

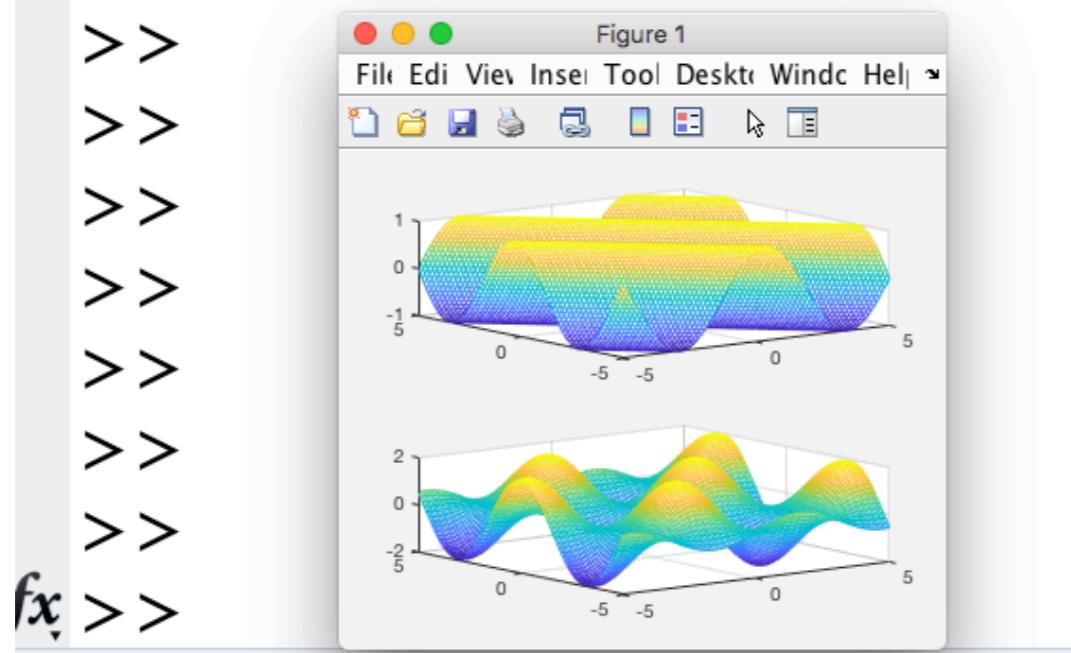


# **LetterCore38**



請完成mesh指令，繪製  $f(x, y) = \sin(x + y) + \sin(x - y)$  的立體圖

```
n = 100;  
a = linspace(-5,5,n);  
X = [REDACTED];  
Y = [REDACTED];  
[REDACTED]  
mesh(a,a,sin(X+Y))  
subplot(2,1,2)  
mesh(a,a,[REDACTED])
```



本題答題

請完成下列程式，印出  $x^3 - 2x + 1$  的一階導數、二階導數及三階導數

Command Window

```
x = input('x');
s= diff(x^3-2*x+1)
for i = 2: 3
    s = diff(s)
end
```

s =

$3*x^2 - 2$

s =

$6*x$

s =

$f\ddot{x}$  6

請將代表  $x^3 - 2x + 1$   
的一階導數、二階導數與三階導數的字串分別儲存於cell， s{1}、s{2}與s{3}

```
x = sym('x');
s{1}= diff(x^3-2*x+1);
display(s{1})
for i = 2: 3
    s{i} = [REDACTED];
    display(s{i})
end
```

ans =

$3*x^2 - 2$

ans =

$6*x$

ans =

6

請將代表  $x^3 - 2x + 1$

的一階導數、二階導數與三階導數的inline函數分別儲存於cell， f{1}、f{2}與f{3}

```
>>
x = sym('x');
s{1} = diff(x.^3-2*x+1);
for i = 2: 3
    s{i} = [redacted];
end
for i = 1 : 3
    f{i} = [blue];
    display(f{i})
end
```

=

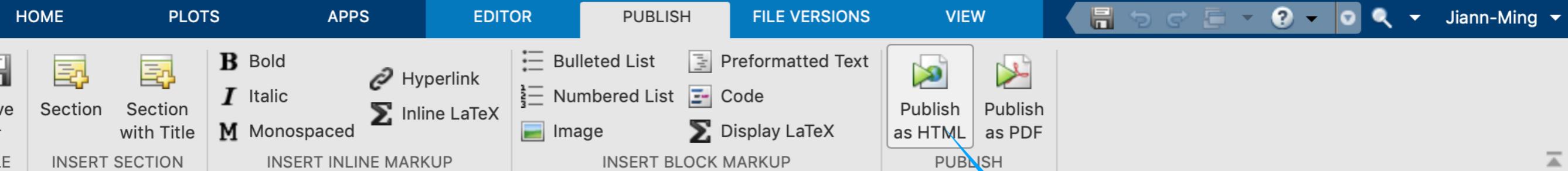
```
|   Inline function:
|   (x) = x.^2.*3.0-2.0
```

=

```
|   Inline function:
|   (x) = x.*6.0
```

=

```
|   Inline function:
|   (x) = 6.0
```



File navigation bar: MATLAB Drive / Published / third\_derivatives.m

**CURRENT FOLDER** (Left pane):

Name
sinch_plot_01.png
sinch_plot.png
sinch_plot.html
demo_hagan_01.png
demo_hagan.png
demo_hagan.html
third_derivatives

**COMMAND WINDOW** (Bottom pane):

New to MATLAB? See resources for Getting Started.

>> Directory

**WORKSPACE** (Bottom-left pane):

NAME	VALUE	SIZE	CLASS
ans	'/MATLAB ... 1x52	char	
f	1x3 cell	1x3	cell
i	3	1x1	double
s	1x3 cell	1x3	cell
x	1x1 sym	1x1	sym

Mail to cite to teaching assistant

[https://matlab.mathworks.com/users/  
\*jmwu@mail.ndhu.edu.tw\*/Published/  
third\\_derivatives/  
index.html](https://matlab.mathworks.com/users/jmwu@mail.ndhu.edu.tw/Published/third_derivatives/index.html)

Your matlab user id

Published directory

matlab.mathworks.com

MATLAB Online R2019b 喜好項目 Publish and Share MATLAB Code

```
syms x;
s{1} = diff(x.^3-2*x+1);
for i = 2:3
    s{i} = diff(s{i-1});
end
for i = 1:3
    f{i} = inline(s{i});
    display(f{i});
end
```

=

Inline function:  
(x) = x.^2.\*3.0-2.0

=

Inline function:  
(x) = x.\*6.0

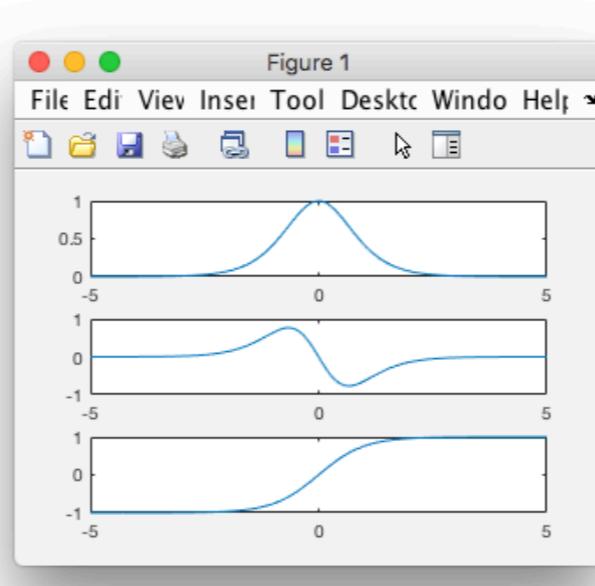
=

Inline function:  
(x) = 6.0

Published with MATLAB® R2019b

請以plot指令繪製 $f\{i\}$ 所代表的inline函數，plot的兩個輸入參數分別為向量a，以及將向量a代入函數 $f\{i\}$ 的結果

```
>> x = sym('x');
s{1} = diff(tanh(x));
for i = 2: 3
    s{i} = [REDACTED];
end
a = linspace(-5,5);
for i = 1 : 2
    f{i} = [REDACTED];
    subplot(3, 1, i)
    [REDACTED]
end
subplot(3,1,3)
plot(a, tanh(a))
```



本題求 $\tanh(a+da)$ 的泰勒展開，展開的項數為三，請將第一項的內容儲存在變數app，並在迴圈中將da的k次方項加入變數app中

```
K = 2; a = pi/4; da = 10^-2;  
x = sym('x');  
s{1} = diff(tanh(x));  
for i = 2: K  
    s{i} = diff(s{i-1});  
end  
fprintf('tanh(a+da) = %15.14f\n',tanh(a+da));  
app = [redacted];  
for k = 1 : K  
    f{k} = inline(s{k});  
    app = app + [redacted];  
end  
fprintf('app = %15.14f\n', app);
```

tanh(a+da) = 0.66145622234088  
app = 0.66145616633167

$$f(a + \delta) \sim f(a) + f'(a)\delta + \frac{1}{2!}f''(a)\delta^2$$

本題求 $\tanh(a+da)$ 的泰勒展開，展開的項數為五，請將第一項的內容儲存在變數app，並在迴圈中將da的k次方項加入變數app中

```
K = 5; a = pi/4; da = 10^-2;
x = sym('x');
s{1} = diff(tanh(x));
for i = 2: K
    s{i} = diff(s{i-1});
end
fprintf('tanh(a+da) = %15.14f\n',tanh(a+da));
app = 0;
for k = 1 : K
    f{k} = inline(s{k});
    app = app + f{k}*da^k;
end
fprintf('app      = %15.14f\n', app);
tanh(a+da) = 0.66145622234088
app      = 0.66145622234724
```

本題求 $\tanh(a+da)$ 的泰勒展開，展開的項數為七，請將第一項的內容儲存在變數app，並在迴圈中將da的k次方項加入變數app中

```
K = 7; a = pi/4; da = 10^-2;
x = sym('x');
s{1} = diff(tanh(x));
for i = 2: K
    s{i} = diff(s{i-1});
end
fprintf('tanh(a+da) = %15.14f\n',tanh(a+da));
app = 0;
for k = 1 : K
    f{k} = inline(s{k});
    app = app + f{k}*da^k;
end
fprintf('app      = %15.14f\n', app);
tanh(a+da) = 0.66145622234088
app      = 0.66145622234088
```

# Exercise taylor\_ex2

$$f(x) = \exp(-x^2)$$

$$f(a + \delta) \sim f(a) + \sum_{k=1}^K \frac{1}{k!} f^k(a) \delta^k$$

$$a = \frac{1}{2}, \quad \delta = 0.01$$

$$f(a + \delta) = ?$$

1. Revise the Matlab script at page 20 to find Taylor expansions of  $e^{-x^2}$  with K = 6
2. Set a = 1/2 and delta = 0.01  
Substitute a + delta to the Taylor expansions