Matlab
Newton's method

START

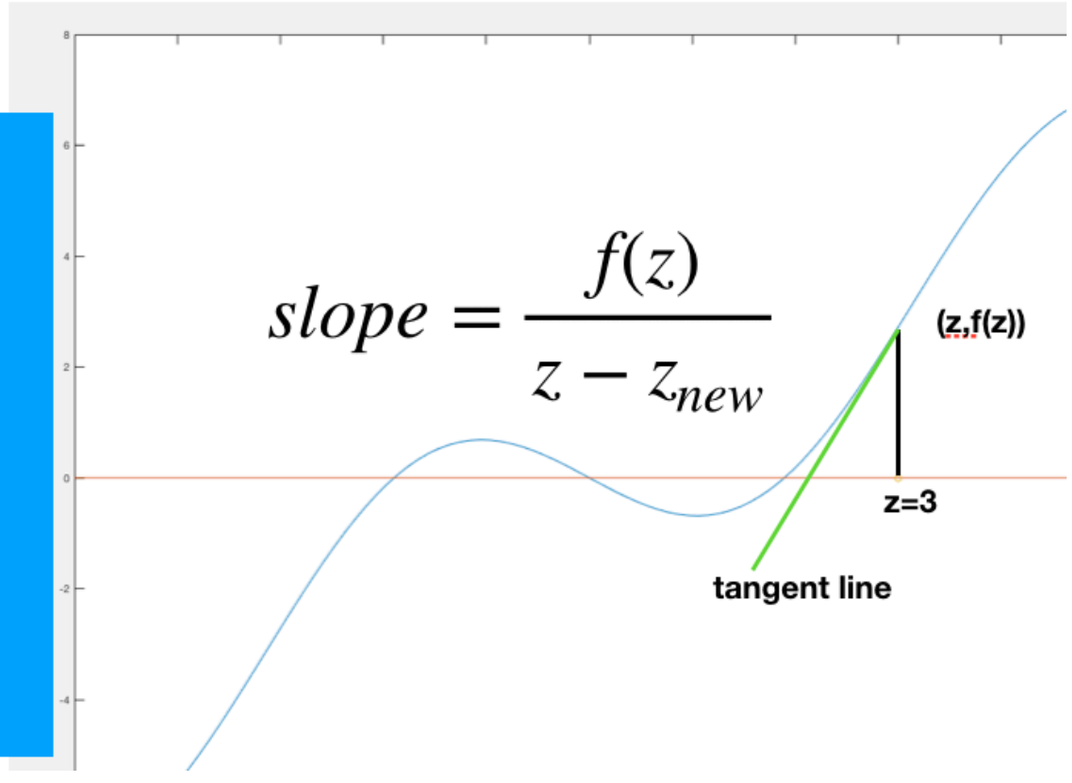
```
s = 'x - 2 * sin(x)'
f=inline(s)
t = diff(str2sym(s))
df = inline(t)
z= rand;
```

while abs(f(z)) > 10^-6

```
display(z)
slope = df(z);
if abs(slope) >10^-6
      z =  z - f(z)/slope;
else
      break
end
```

fz = f(z)

END



For example, we might wish to solve equations of the form

$$x - 2^{-x} = 0,$$
$$\mathrm{e}^x - x^2 + 3x - 2 = 0,$$
$$x \cos x + 2x^2 + 3x - 2 = 0.$$

START

```
s = 'x - 2 * sin(x)'
f=inline(s)
t = diff(str2sym(s))
df = inline(t)
z= rand;
```

while abs(f(z)) > 10^-6

```
display(z)
slope = df(z);
if abs(slope) >10^-6
      z =  z - f(z)/slope;
else
      break
end
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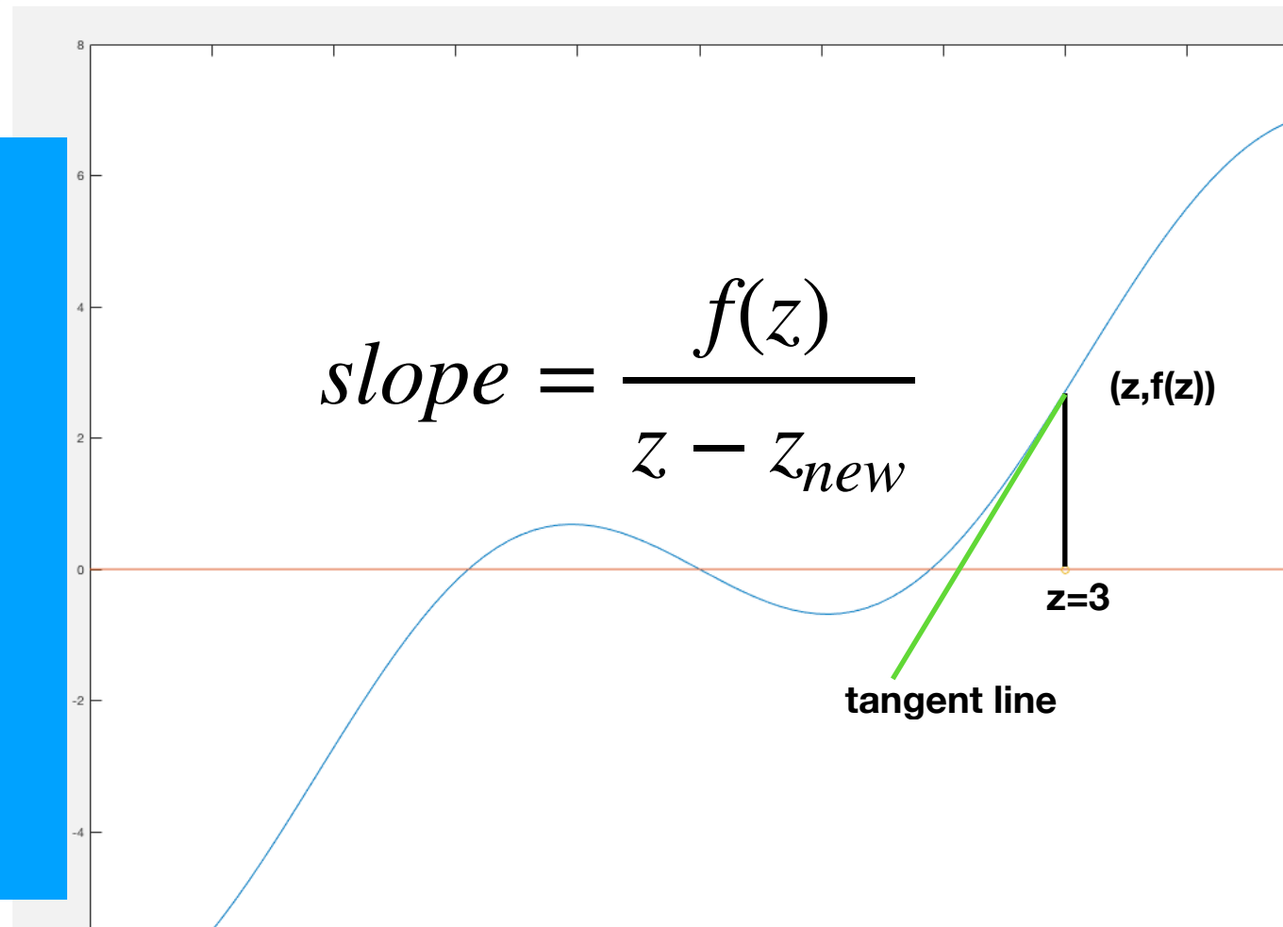
fz = f(z)

END



https://
matlab.mathworks.com/users/
jmwu@mail.ndhu.edu.tw/
Published/demo_newton/
index.html

Exercise

- Write a matlab script to implement the flowchart for solving a nonlinear function by the Newton method
- Try to solve your nonlinear function
- Publish your codes

S為字串，代表自變數為x的函數運算式，matlab提供fsolve指令，求非線性函數最佳解．fsolve的指令參數包括s字串以及初始值，fsolve的輸出參數為所求得的根．fsolve的用途相當廣泛，包括非線性系統求解，是maltab非常重要的解題指令．

```
>> s='x.^2-5*x+6'; fsolve(s,1.5)

Equation solved.

fsolve completed because the vector of function values is near zero
as measured by the default value of the function tolerance, and
the problem appears regular as measured by the gradient.

<stopping criteria details>


ans =

   1.999999976777320
```

請完成字串s的設定，解 $x^2 - 2x - 3 = 0$ **，並設定fsolve的初始值為0.5**

```
s = '                ';
z = fsolve(s,    )
```

Equation solved.

fsolve completed because the vector of function values is near zero as measured by the default value of the function tolerance, and the problem appears regular as measured by the gradient.

<stopping criteria details>

```
z =

  -1.000000000000127
```

fsolve指令可以使用將符號“@”置於函數名稱之前的輸入參數．本題定義myfun函數，實作 $f(x) = x^2 - 2x - 3$ ．本題以fsolve指令求根，請在myfun前加@符號

```
1   function demo()
2       z = fsolve(          , 0.5)
3
4   function ans = myfun(x)
5       ans =                ;
```

Command Window

Equation solved.

fsolve completed because the vector of function values is near zero
as measured by the default value of the function tolerance, and
the problem appears regular as measured by the gradient.

<stopping criteria details>

z =

-1.000000000000127

fx

第二行將fsolve的輸出值存在變數z中，如何判斷變數z所存的內容代表函數根呢？請在第三行印出z的函數值，如果函數絕對值接近0，則z的內容代表函數根，fsolve求解成功

```
1    function demo()
2      z = fsolve(          , 0.5);
3      display(          )
4
5    function ans = myfun(x)
6      ans =                ;
```

本題答題

Command Window

```
>> untitled

Equation solved.

fsolve completed because the vector of function values is near zero
as measured by the default value of the function tolerance, and
the problem appears regular as measured by the gradient.

<stopping criteria details>

    5.089262344881718e-13

fx >>
```

fsolve可以使用輸入參數optimoptions，設定不同的求解方法、停止條件或控制訊息列印，本題使用optimoptions關閉(off)訊息列印．請在第三行將hc設定為判斷z的函數絕對值是否小於10^-6的布林值，hc也可解讀為代表判斷z是否接近0的布林值

```
1   function demo()
2     z = fsolve([redacted], 0.5, [redacted]('fsolve', 'Display', 'off'));
3     hc = [redacted]
4
5   function ans = myfun(x)
6     ans = x^2 - 2*x - 3;
```

Command Window

```
>> untitled

hc =

  logical

   1

fx >>
```

請在第二行使用inline指令，將f指定為可以計算 $x^2 - 2x - 3$ 對應值的運算函數，並在第五行將hc設定為判斷z的函數絕對值是否小於10^-6，hc代表判斷z是否接近0的布林值．本題執行結果hc為true，fsolve求解成功

本題答題

```
1 -     s = 'x^2 - 2*x - 3';
2 -     f = ________;
3 -     z = fsolve(s, 0.5, ________('fsolve', 'Display', 'off'));
4 -     hc = ________
5
```

Command Window

```
>> untitled

hc =

  logical

   1

fx >>
```

本題解 $x^3 - 2x - 5 = 0$

請設定字串變數s，並在第三行設定初始值為3，以指令fsolve求解．本題執行結果hc為true，fsolve求解成功

本題答題

```
1 —     s = '                    ';
2 —     f =          ;
3 —     z = fsolve(s,  , optimoptions('fsolve', 'Display', 'off'))
4 —     hc = abs(f(z)) < 10^-6
5
```

```
Command Window
>> untitled

z =

   2.094551481542352


hc =

  logical

   1
```

本題解 $x - 2sin(x) = 0$

請設定字串變數s，並在第三行設定初始值為3，以指令fsolve求解．本題執行結果hc為true，fsolve求解成功

```
1 —     s = '[redacted]';
2 —     f = [redacted];
3 —     z = fsolve(s, [redacted], optimoptions('fsolve', 'Display', 'off'))
4 —     hc = abs(f(z)) < 10^-6
5
```

本題答題

```
Command Window
>> untitled

z =

   1.895494281543037


hc =

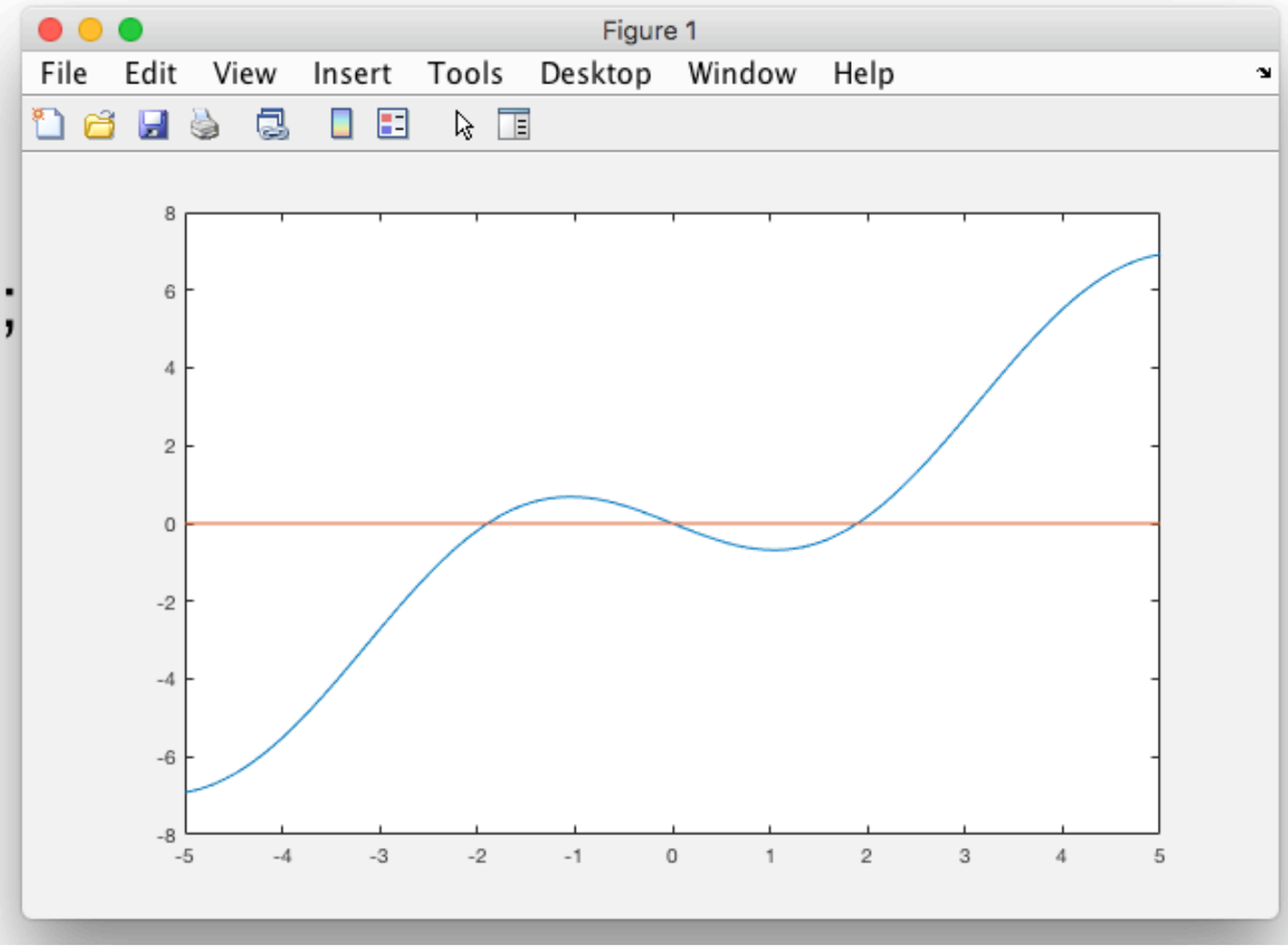
  logical

   1

fx >>
```

請在第二行使用inline指令，將f設定為相對字串s的運算函數
第四行使用plot繪製運算函數f在-5與5間的函數圖
請在第五行使用hold on指令，固定figure，採用疊加方式繪製橫軸

```
>>
 s = 'x- 2*sin(x)';
 f =          ;
 a = linspace(-5,5);
 plot(        );
      ;
 plot(a, a*0)
fx >>
```



字串s代表 $x - 2sin(x)$ 對應函數

請在第二行使用str2sym將字串s轉換為符號並且以diff求對應函數 $x - 2sin(x)$ 的導數，運算結果儲存在符號變數df中

```
s = 'x- 2*sin(x)';
df =
```

```
df =

1 - 2*cos(x)
```

字串s代表 $x-2sin(x)$ 對應函數

請在第二行先使用str2sym將字串s轉換為符號，再使用diff求對應函數

$x-2sin(x)$ 的導數，最後再以inline將結果轉換為可運算函數，儲存在函數變數df中

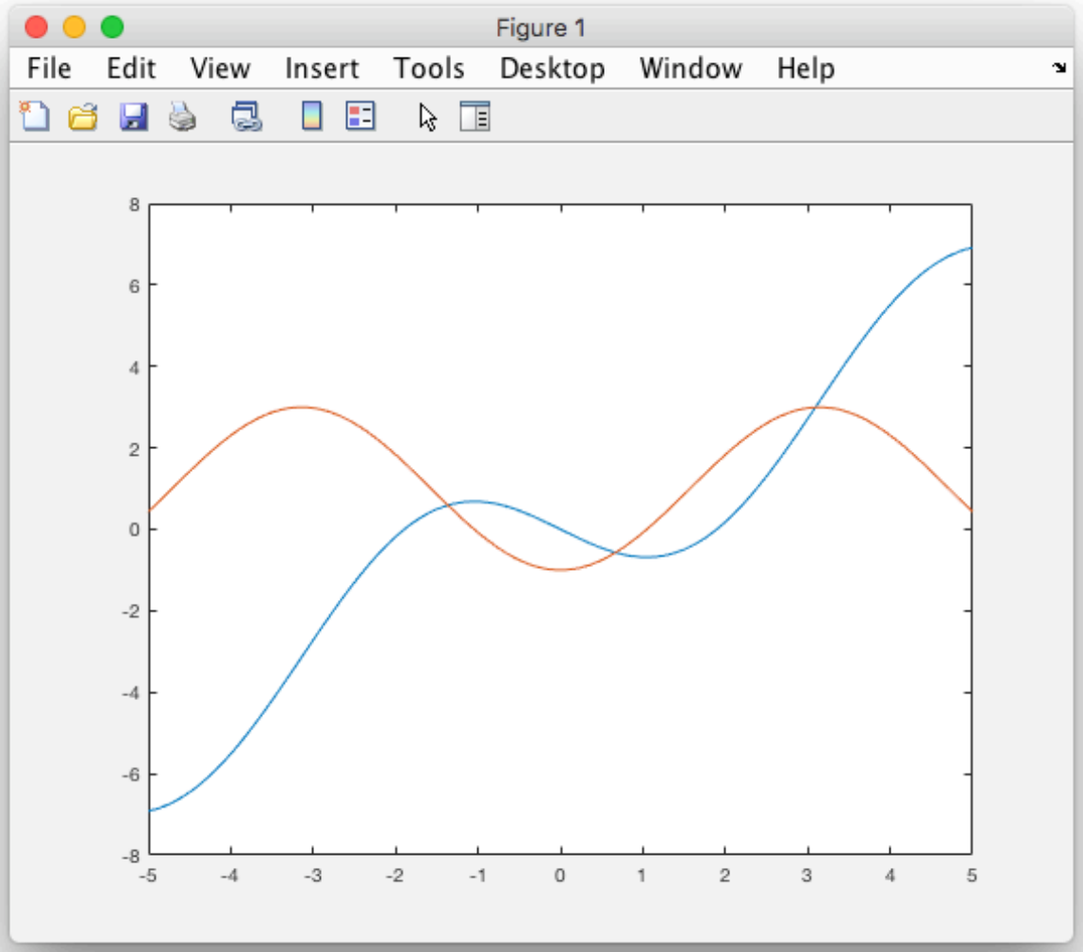
```
s = 'x- 2*sin(x)';
df =
```

```
df =

    Inline function:
    df(x) = cos(x).*-2.0+1.0
```

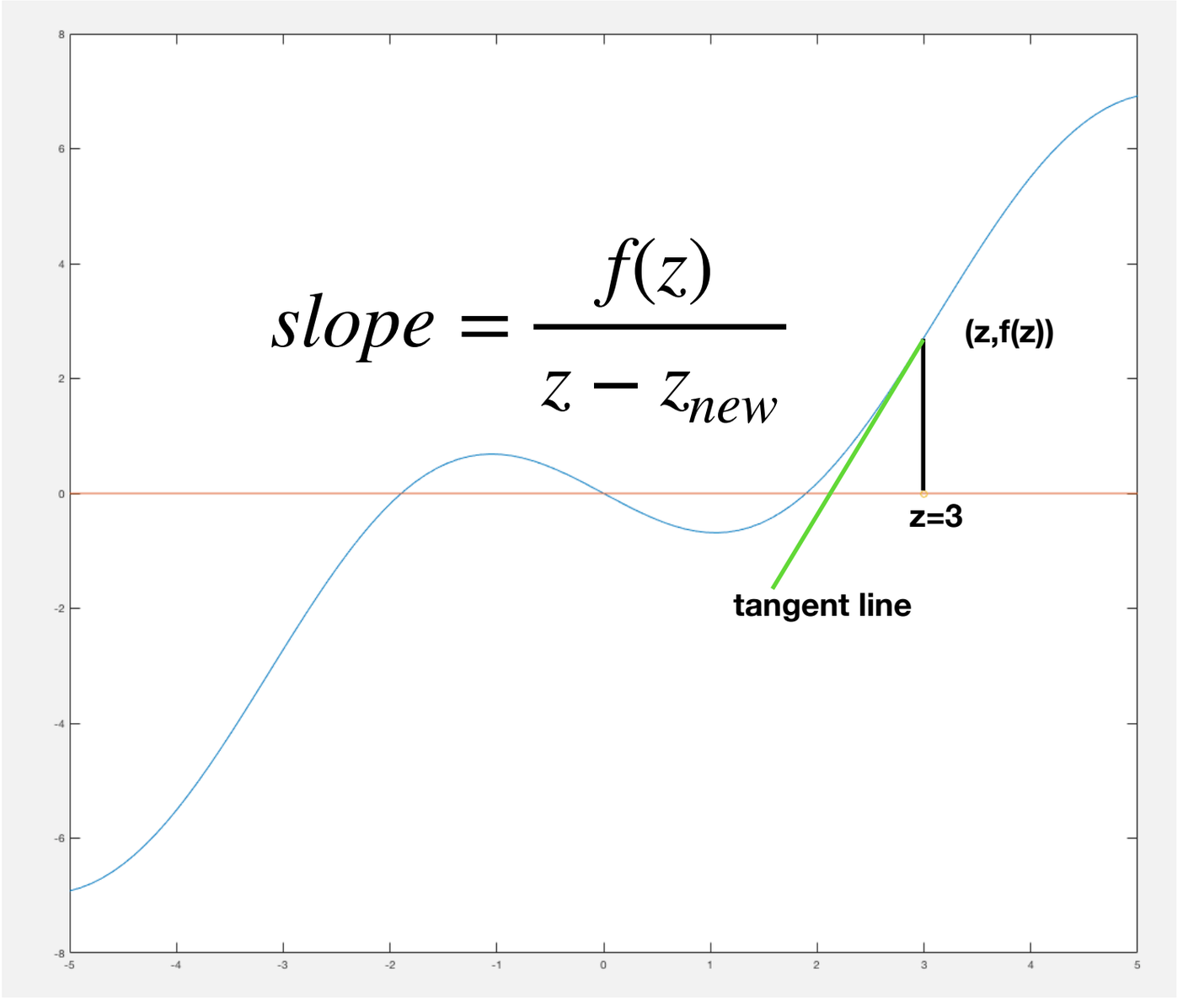
請完成對應函數 $x - 2sin(x)$ 及其導函數之繪製

```
s = 'x- 2*sin(x)';
f = [redacted];
df = [redacted];
a = linspace(-5, 5);
plot(a, f(a));
hold on;
plot([redacted]);
>>
>>
```



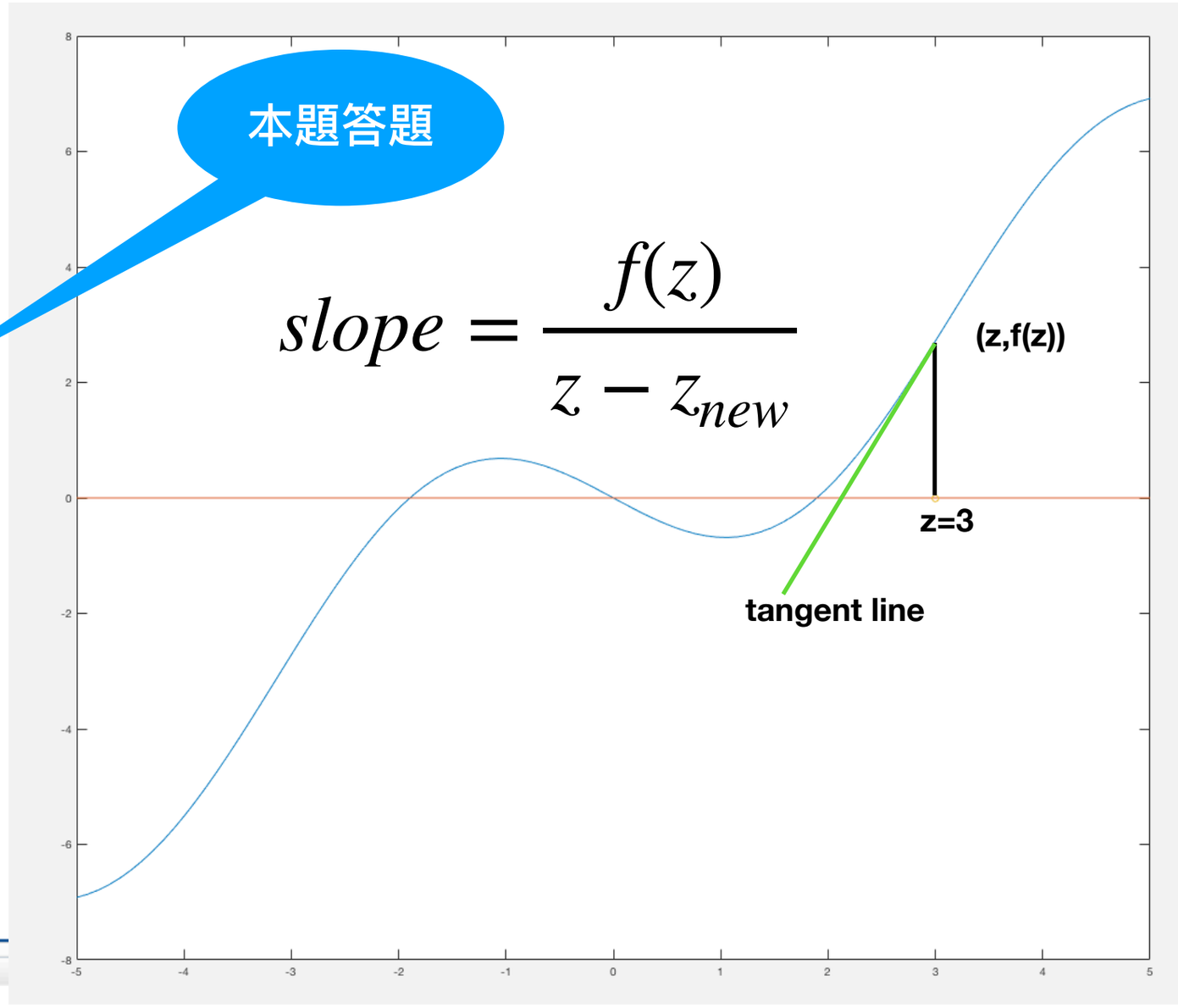
本題f與df分別代表可運算函數與導數，第四行將z值設定為3，請在第五行計算圖中綠色切線(tangent line)的斜率，並儲存在變數slope，在條件指令中更新z值，使新的z值代表綠色切線與紅色橫軸的交叉位置

```
s = 'x- 2*sin(x)';
f =          ;
df =                                    ;
z = 3;
slope =       ;
if  abs(slope) > 10^-6
        z =
end
```



牛頓法使用迭代運算求非線性函數的根．請在第六行加入while迴圈，迴圈執行的進入條件為z的函數絕對值大於10^-6

```
1  s = 'x- 2*sin(x)';
2  f =                                    ;
3  df =                                                    ;
4  z = 3;
5  slope =                ;
6                                                          
7      if  abs(slope) > 10^-6
8          z =                                          ;
9      end
10 end
11 display(z)
```



Command Window

```
z =

   1.895494701250431
```

當斜率為0或接近0，第七行的if指令條件不成立，避免更新公式的分母為0或接近0，請在第十行以break指令中斷迴圈執行，並在第十四行印出函數值

```
1   s = 'x- 2*sin(x)';
2   f = [redacted];
3   df = [redacted];
4   z = 3;
5   slope = [redacted];
6   while [redacted]
7       if  abs(slope) > 10^-6
8           z = [redacted];
9       else
10          [redacted]
11      end
12  end
13  display(z)
14  display([redacted])
```

本題答題

Command Window

```
1.895494701250431


 7.112662847585938e-07
```