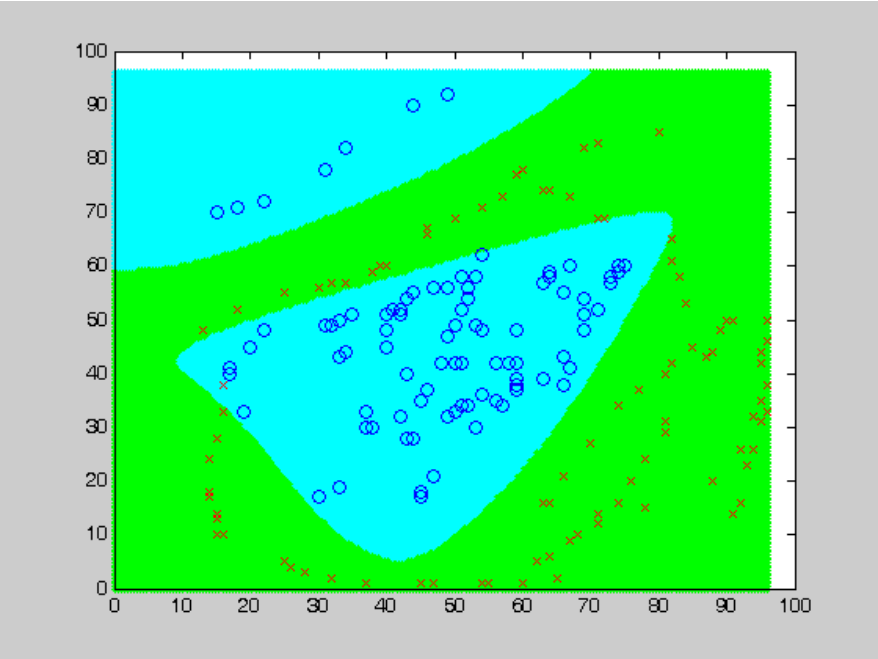
Discriminant Analysis



Separation

- Data Classification
- Linearly separable
- Nonlinearly Separable
- LM-RBF learning for data separation

Points with labels

$$\mathbf{x}[t] \in \mathrm{R}^{\mathrm{d}}$$

$$y[t] \in \{\mathrm{Blue}, \mathrm{Red}\}$$

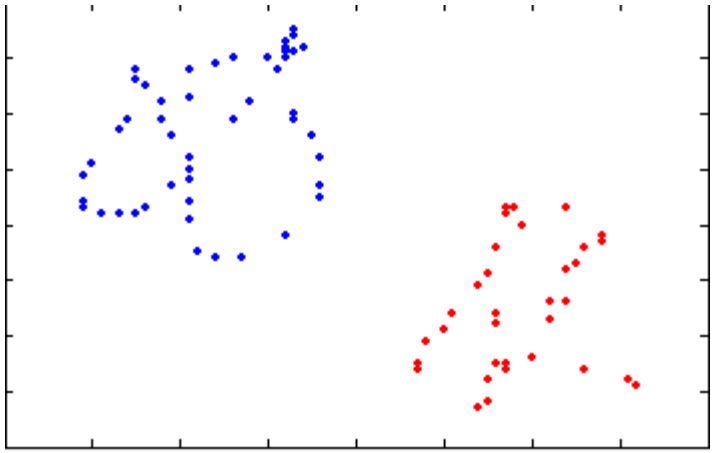
$$or$$

$$y[t] \in \{0, 1\}$$

Example

- d=2
- x[t] : data coordinate
- y[t] : data color

$$x[t] = (0.8\ 0.3)^T, y[t] = 'BLUE'$$



Example

- Iris data iris.txt UCI Machine Learning Repository
- d=4
- y[t] $\in$ {Iris-setosa, Iris-versicolor, Iris-virginica}
- $x[t]=(x_1[t], x_2[t], x_3[t], x_4[t])$

  x_1: sepal length in cm

  x_2: sepal width in cm

  x_3: petal length in cm

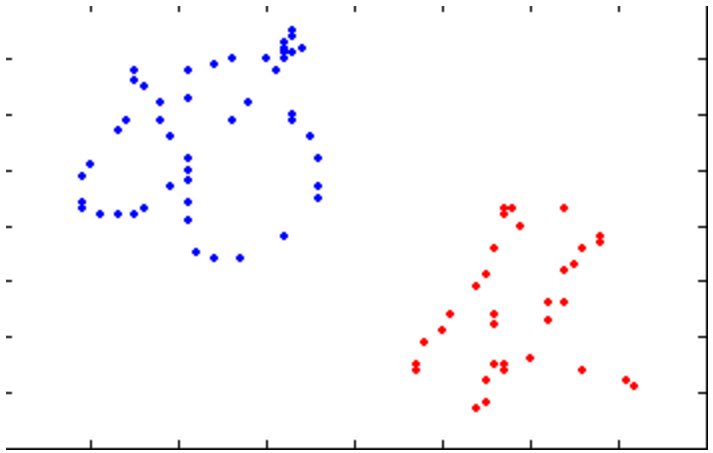
  x_4: petal width in cm

Example

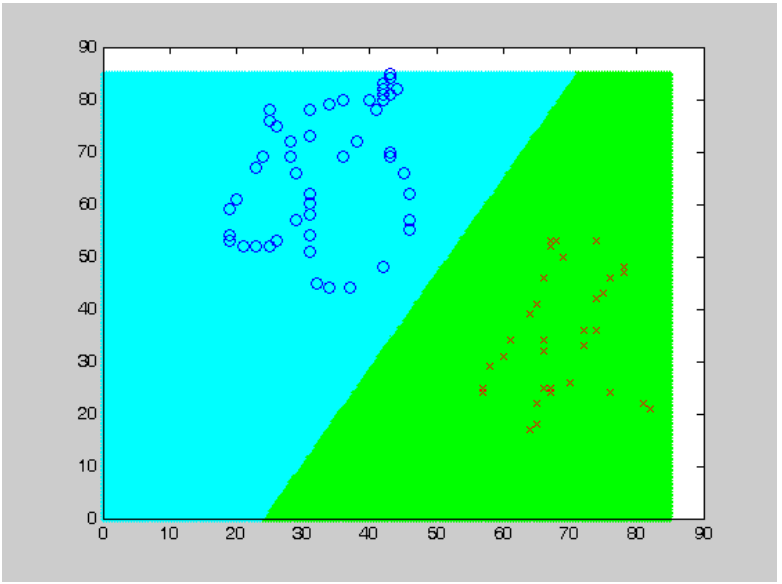
- Wine data
- Breast Cancer Wisconsin (Diagnostic)

Linear Separation

- 2D labeled data



- Linear Separation



Linear Separation

$$y \in \{0,1\}$$

$$f(\mathbf{x};\mathbf{a}) = x_1 a_1 + x_2 a_2 + a_3$$

$$y = 1, \quad \text{if } f(\mathbf{x}) > 0.5$$

$$= 0, \quad \text{otherwise}$$

Over-determined linear system

$$x_1[\mathrm{t}]a_1 + x_2[\mathrm{t}]a_2 + a_3 = y[t]$$

$$t = 1,\ldots, N$$

Least square method

Minimize

$$E(\mathbf{a}) = \frac{1}{N}\sum_{i=1}^{N}\left(\mathbf{x}^T[t]\mathbf{a} - y[t]\right)^2$$

Least square method

1. Let X denote data matrix of size Nxd
2. A=[X ones(N,1)]
3. pinv(A)*y

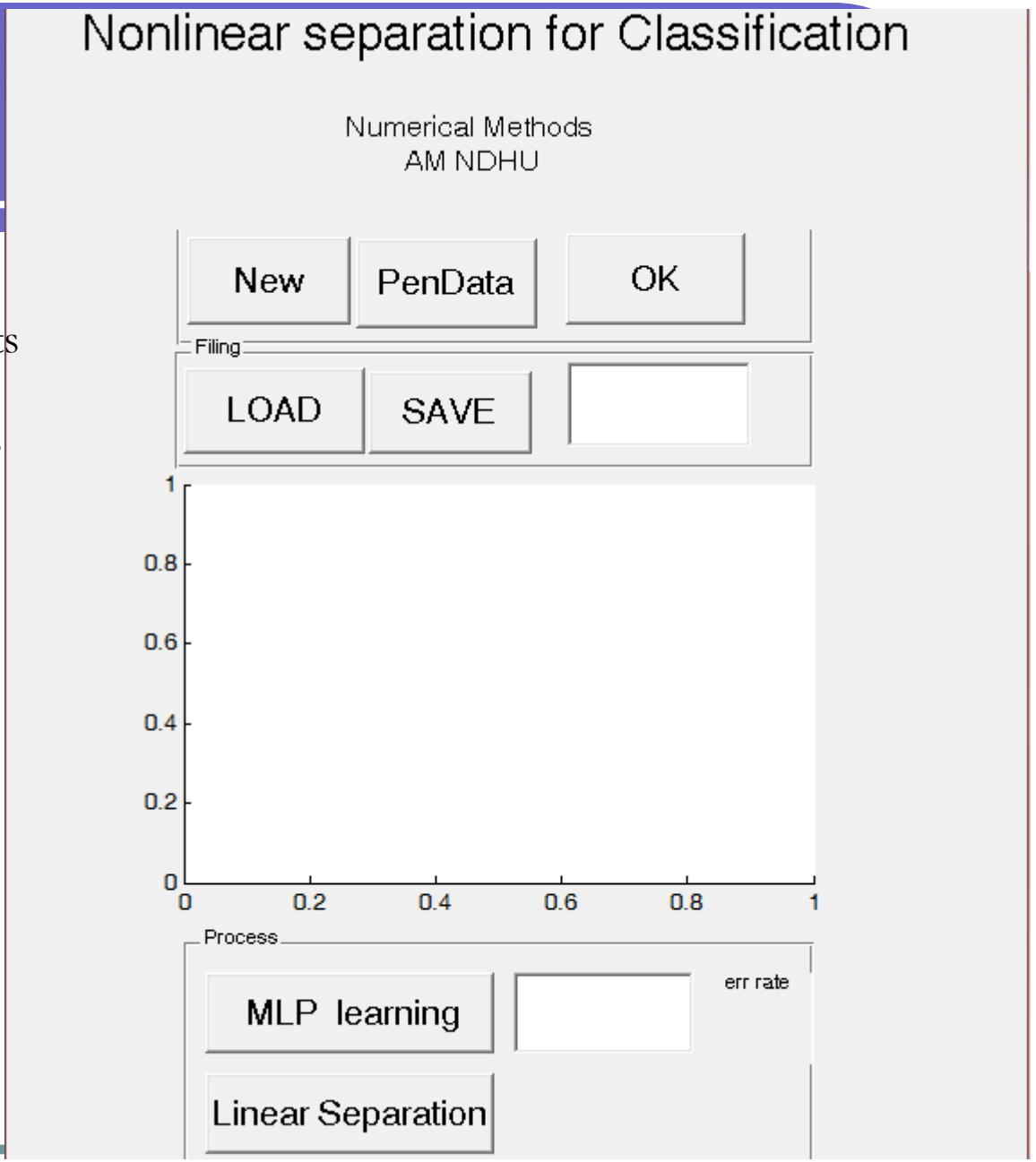
Toolkit for data separation

Toolkit for data separation

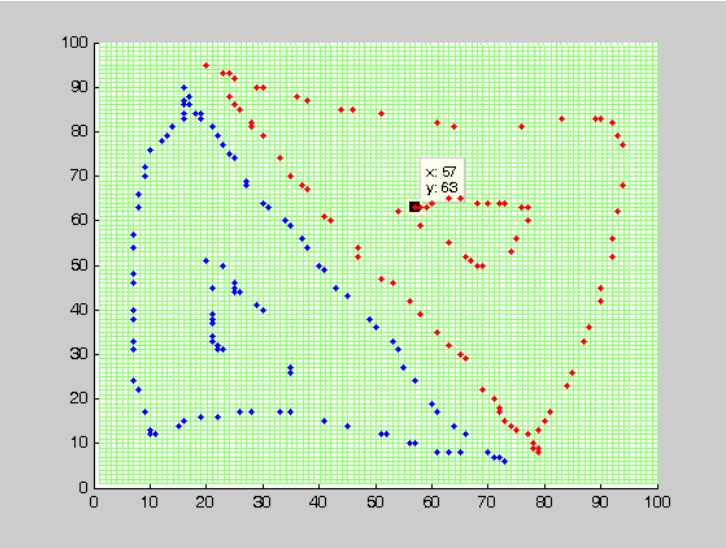
Nonlinear separation for Classification

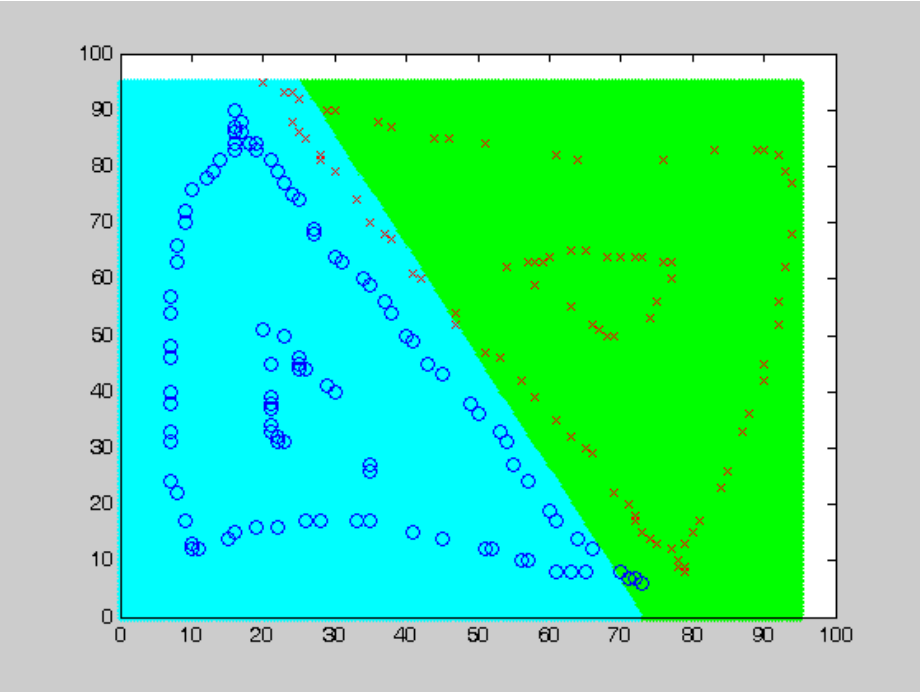
- Press New to start
- Press Pendata to paint blue points
- Press OK to get data
- Press Pendata to paint red points
- Goto step 2
  or press Linear Separation

- Press SAVE to store paired data
- Press LOAD to get paired data

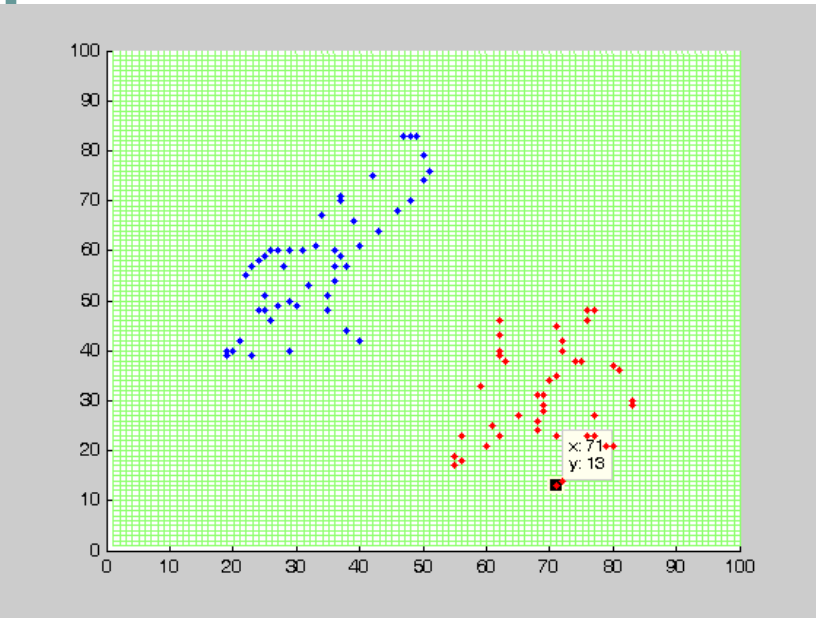


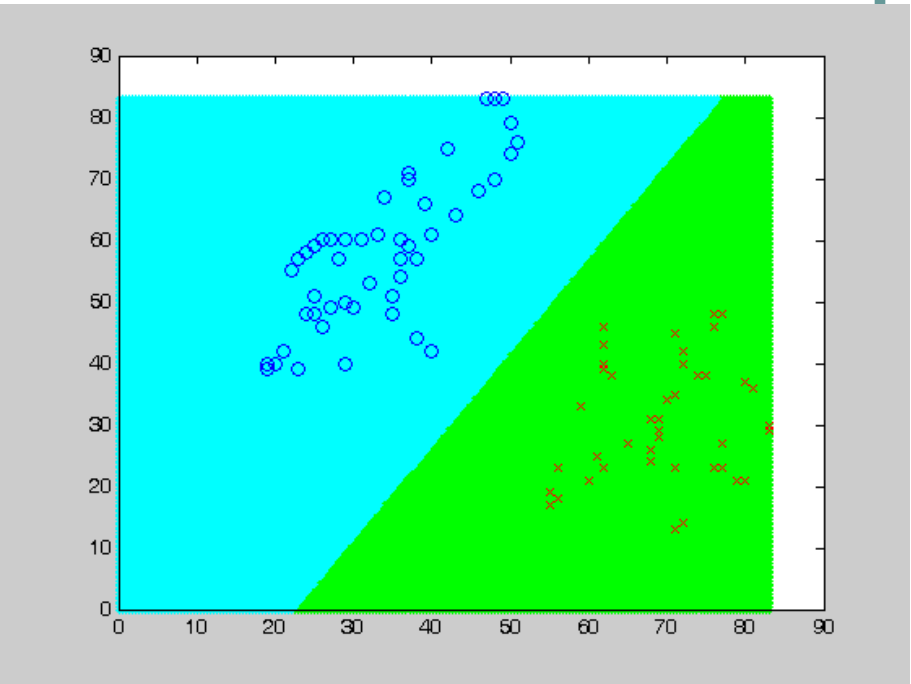
Linearly separable





Linearly separable





Coding

- a=finda(x,y)
  - Given x and y, find vector a to minimize the mean square error
- yhat=x2y(x,a)
  - Map x to y for given a

```
1  function a=finda(x,y)
2 -        b=y;
3 -        A=[x ones(length(b),1)];
4 -        a=pinv(A)*b';
5
```

finda.m

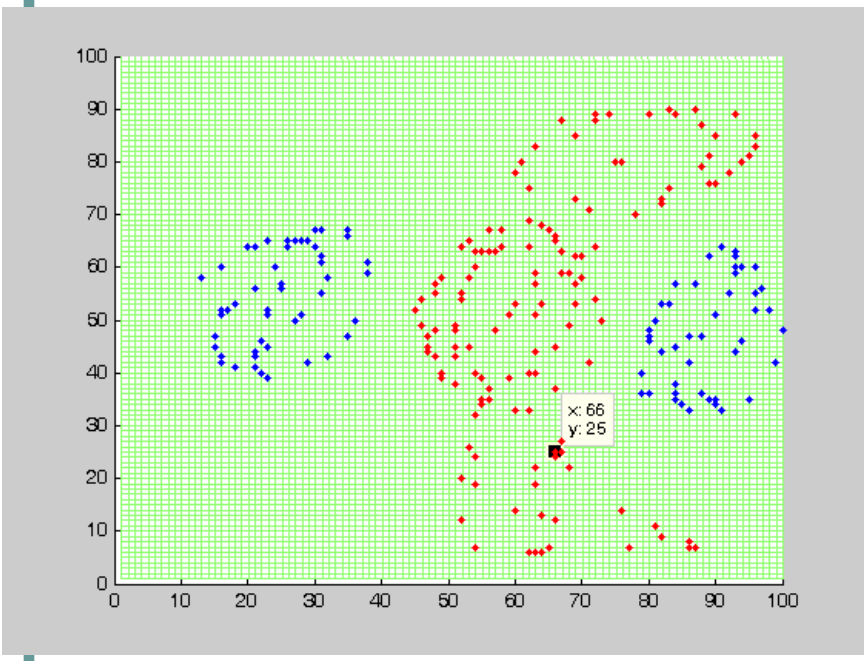
x2y.m

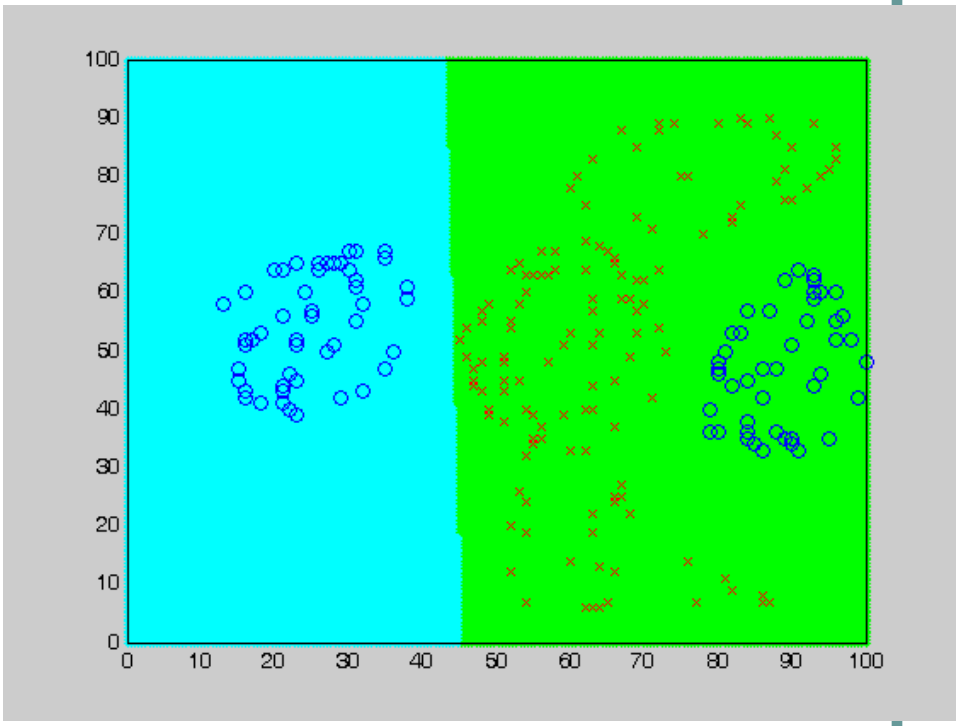
```
1  function yhat=x2y(x,a)
2-         A=[x ones(size(x,1),1)];
3-         yhat=A*a;
4
```

```
349 -       y(ind)=0;
350 -       n=200;x1=linspace(0,max_x,n);x2=x1;
351 -       xx=[repmat(x1',n,1) reshape(repmat(x2,n,1),n*n,1)];
352         %%%%%%%%%%%%%%%%%
353 -       a=finda(x/max_x,y);
354 -       yhat=x2y(x/max_x,a);
355 -       CC=x2y(xx/max_x,a);
356         %ex.m
357
358 -       err=mean(abs((yhat>0.5)-y'))
359         %%%%%%%%%%%%%%%%%
```

Calling finda and x2y

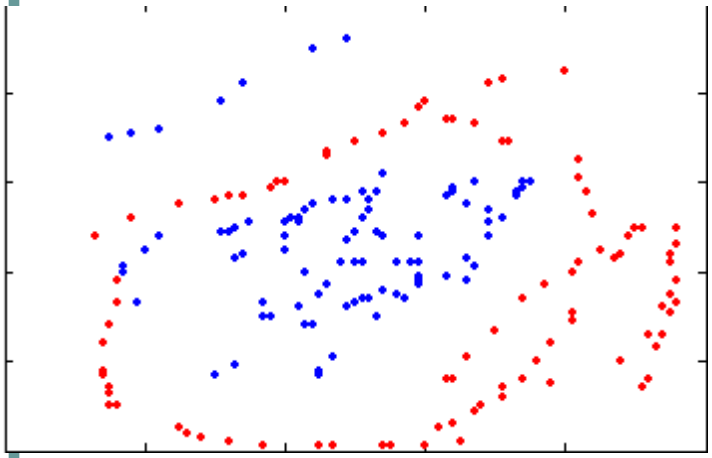
SeparableTest.m

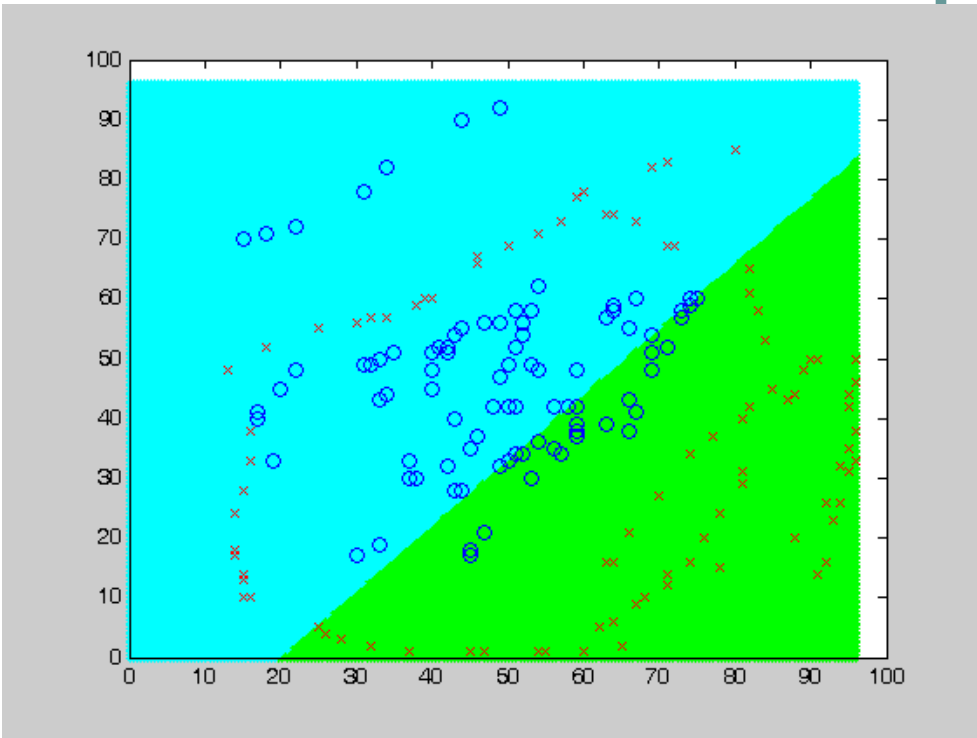

x: 66
y: 25



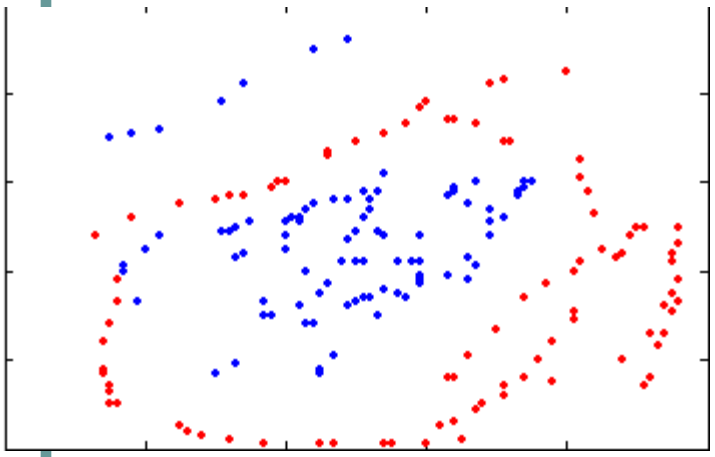
Linearly non-separable

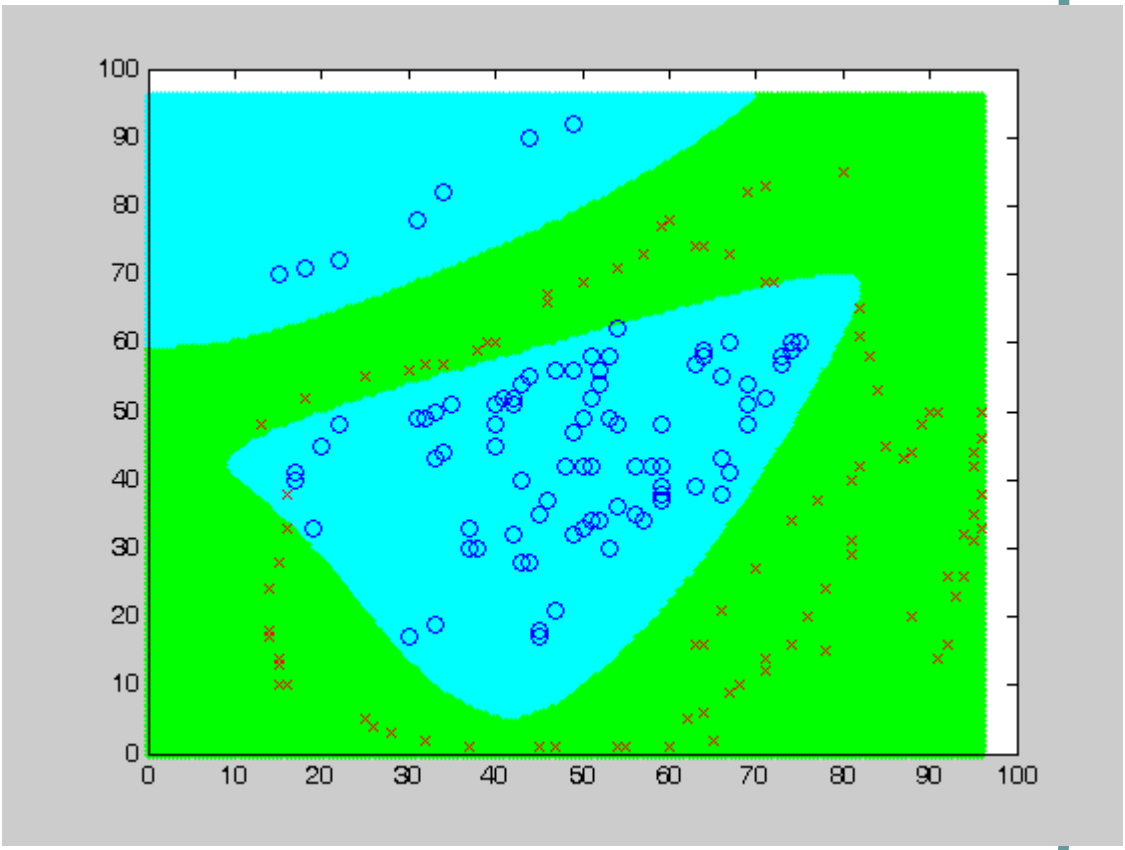
- Non-separable data by linear separation





Non-linearly separable





```
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352         %%%%%%%%%%%%%%%%
353 -       a=finda(x/max_x,y);
354 -       yhat=x2y(x/max_x,a);
355 -       CC=x2y(xx/max_x,a);
356         %ex.m
357
358 -       err=mean(abs((yhat>0.5)-y'))
359         %%%%%%%%%%%%%%%%
```
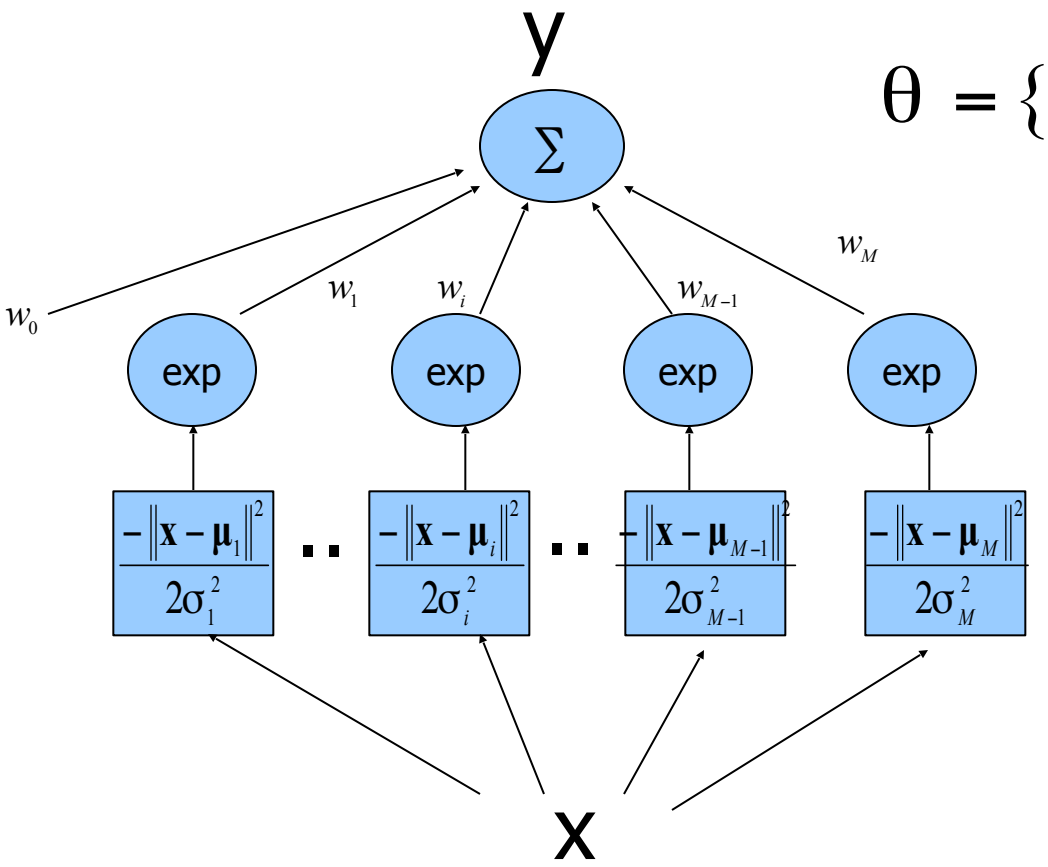

SeparableTest.m

RBF



Network parameter

$$\theta = \{w_i\}_i \cup \{\boldsymbol{\mu}_i\}_i \cup \{\sigma_i\}_i$$

Nonlinear separation by LM-RBF

- Accomplish ex.m
  - It needs to revise lines 353-355 to apply LM learning of RBF
    - Line 353: net=findnet(x,y,M)
    - Line 354: y_hat=x2y(x,net)
    - Line 355: CC=x2y(xx,net)

findnet

- findnet: apply the LM method to minimize the mean square error of approximating desired targets by outputs of an RBF network
- net=findnet(x,y)

Data Structure

- Net
  - Net.u : receptive field Mxd
  - Net.si : variance Mx1
  - Net.w : posterior weights M+1
  - Net.M : number of hidden units
  - Net.d : data dimension

x2y

- Evaluate an RBF network
- y=x2y(x,net)

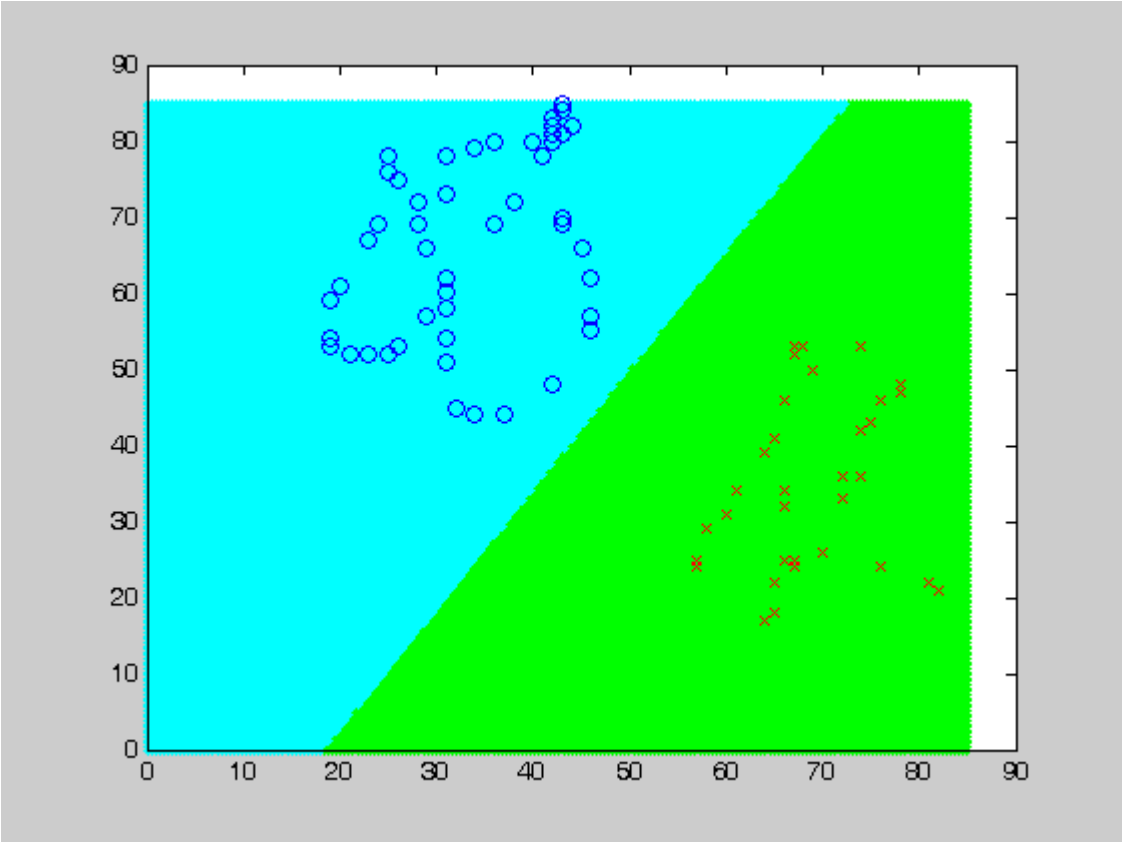
RBF Evaluation

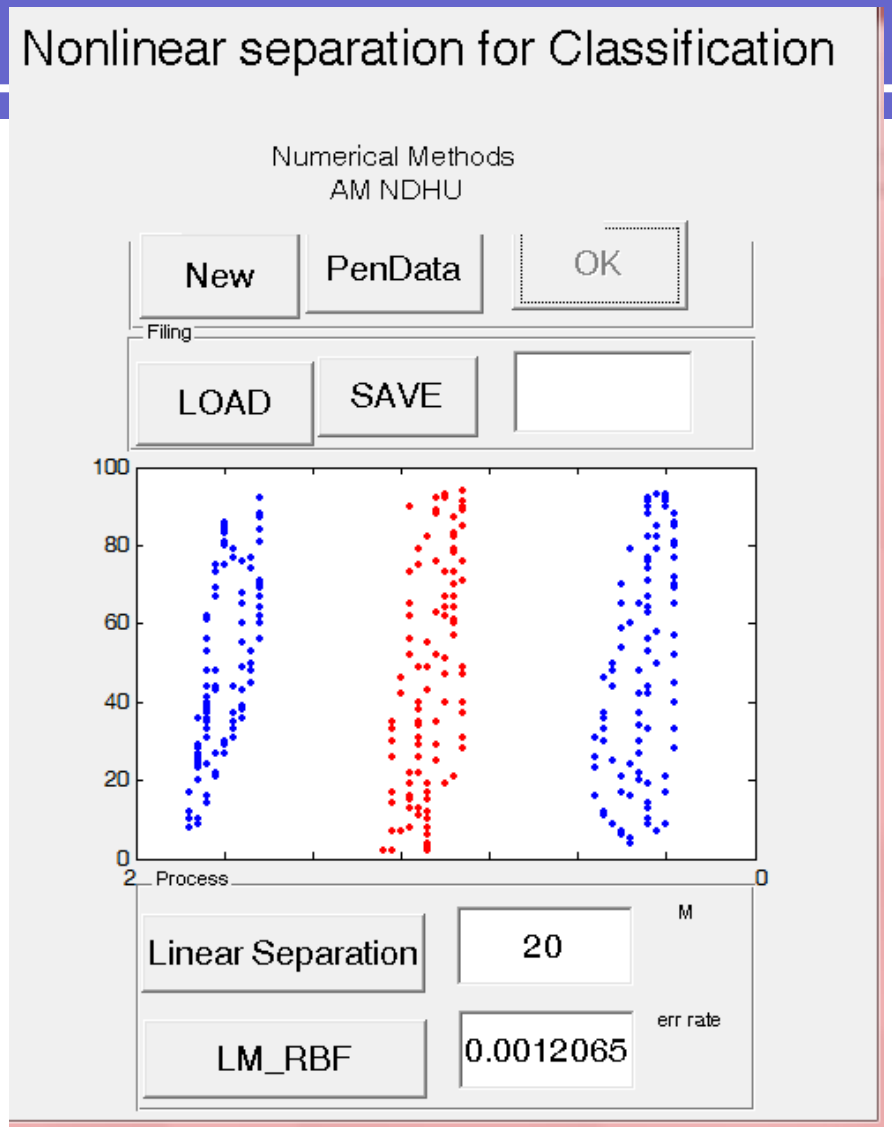
$$y = G(\mathbf{x}\,|\,\theta)$$

$$= w_0 + \sum_{m=1}^{M} w_m \exp\left(-\frac{\|\mathbf{x} - \boldsymbol{\mu}_m\|^2}{2\sigma_m^2}\right)$$

Nonlinear Separation

Linearly separable data




Nonlinear separation for Classification
Numerical Methods
AM NDHU
New
PenData
OK
Filing
LOAD
SAVE
100
80
60
40
20
0
2
Process
0
M
Linear Separation
20
LM_RBF
0.0012065
err rate

LM learning : RBF(20)