Determinant and recurrent mapping

A. The Ikeda map

The Ikeda map[52] is characterized by a nonlinear system,

$$f(x_1, x_2) = \begin{cases} f_1(x_1, x_2) \\ \\ f_2(x_1, x_2) \end{cases}$$

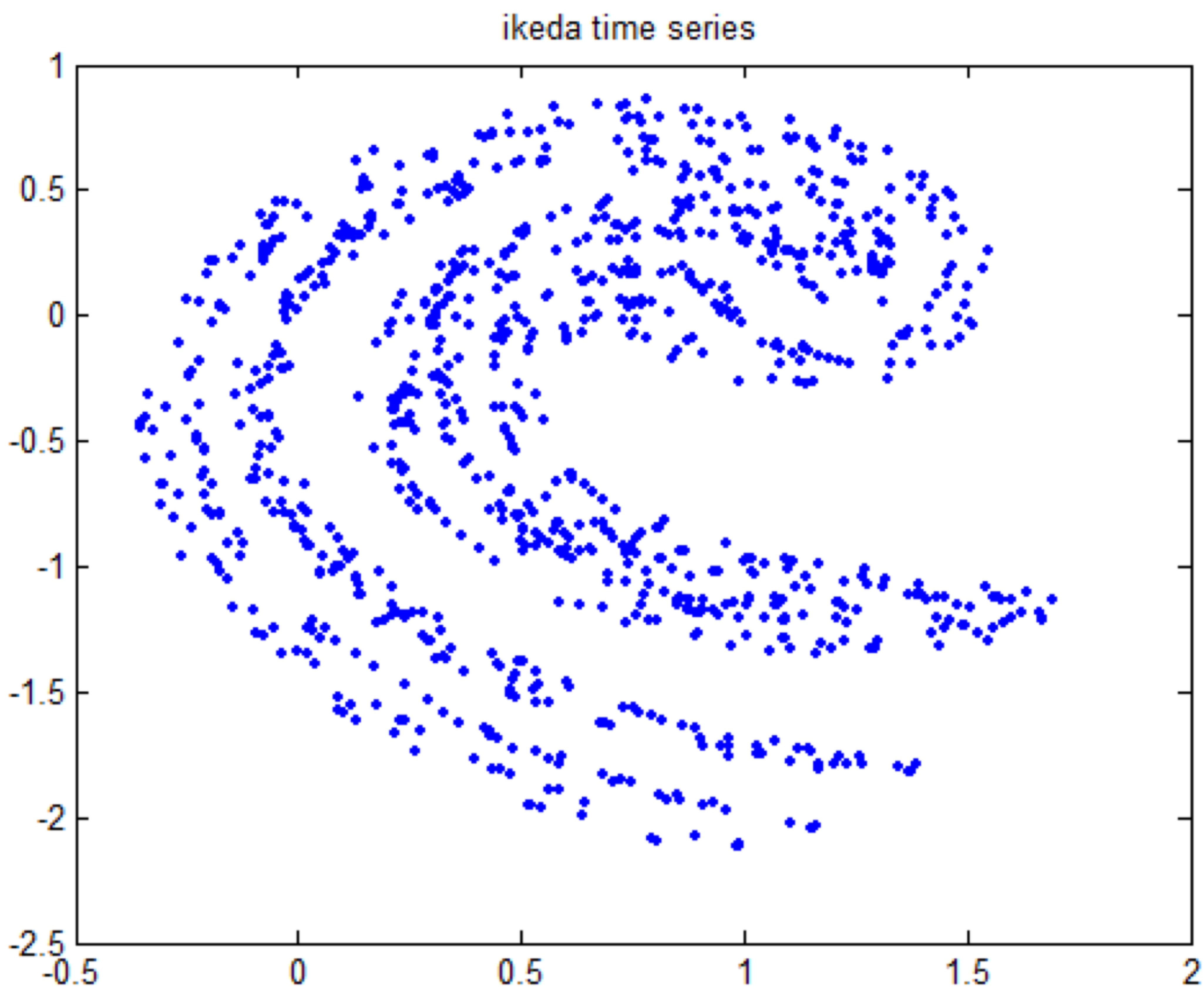
where coordinate functions are given by

$$\begin{aligned} f_1(x_1, x_2) &= R + C_2(x_1 \cos\left(C_1 - C_3/(1 + x_1^2 + x_2^2)\right) - x_2 \sin\left(C_1 - C_3/(1 + x_1^2 + x_2^2)\right)), \\ f_2(x_1, x_2) &= C_2(x_1 \sin\left(C_1 - C_3/(1 + x_1^2 + x_2^2)\right) + x_2 \cos\left(C_1 - C_3/(1 + x_1^2 + x_2^2)\right)), \end{aligned}$$

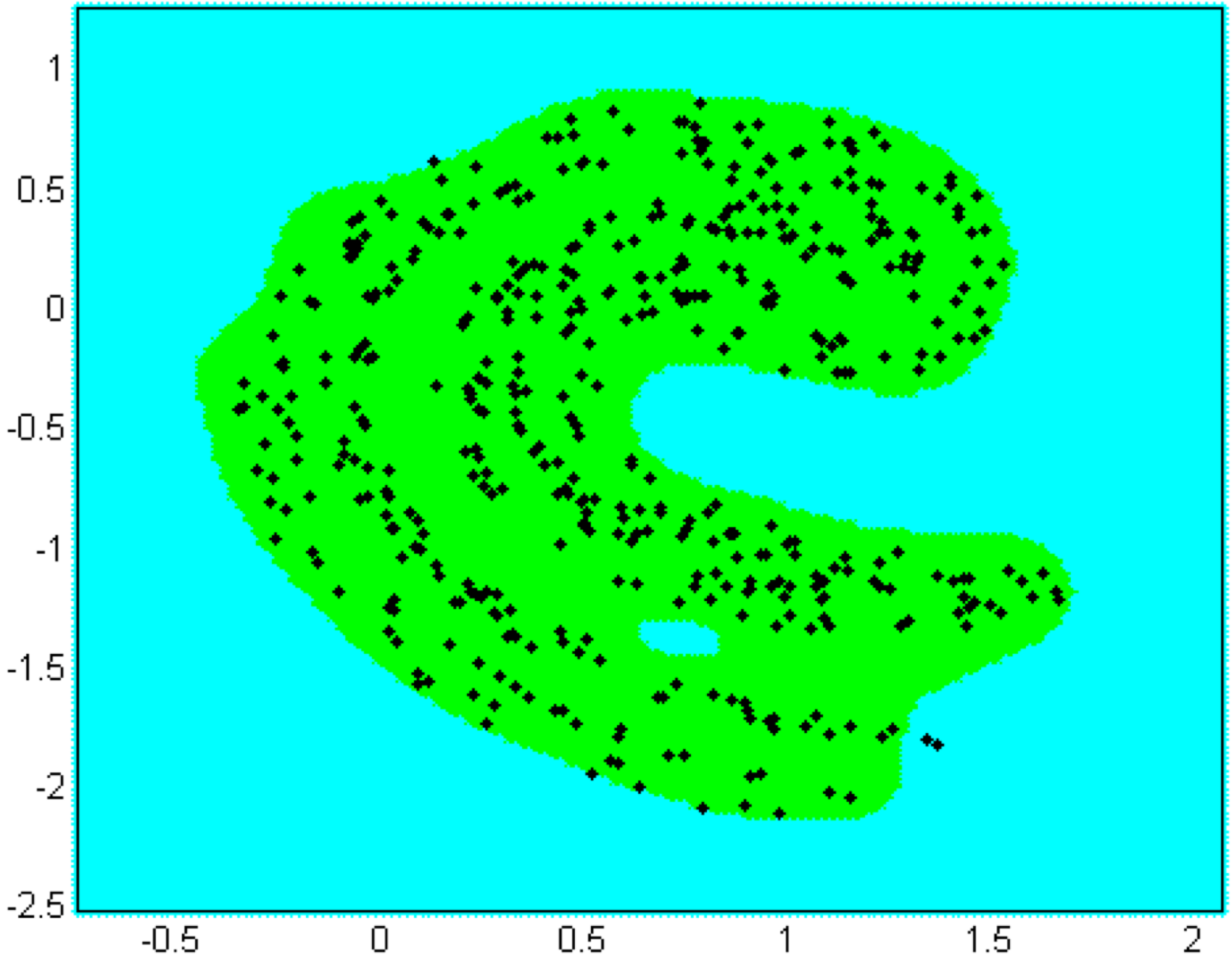
The parameters are given by $R = 1, C_1 = 0.4, C_2 = 0.9$, and $C_3 = 6$.

Let $\mathbf{x}[n] = (x_1[n], x_2[n])$ and $\mathbf{x}[0]$ denote a random initial condition. The Ikeda time series is generated by the first-order recursive function,

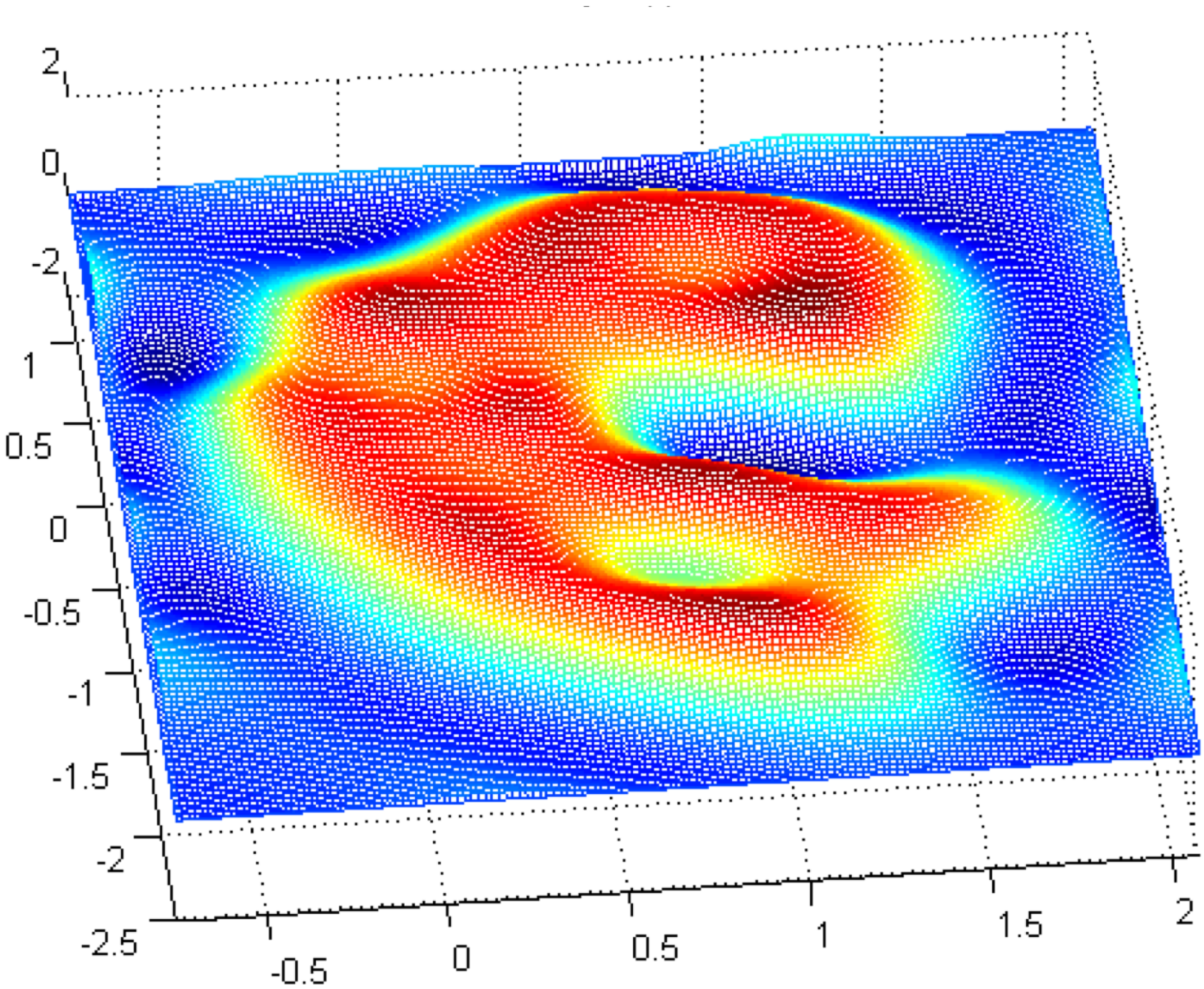
$$\mathbf{x}[n+1] = f(\mathbf{x}[n]). \tag{1}$$

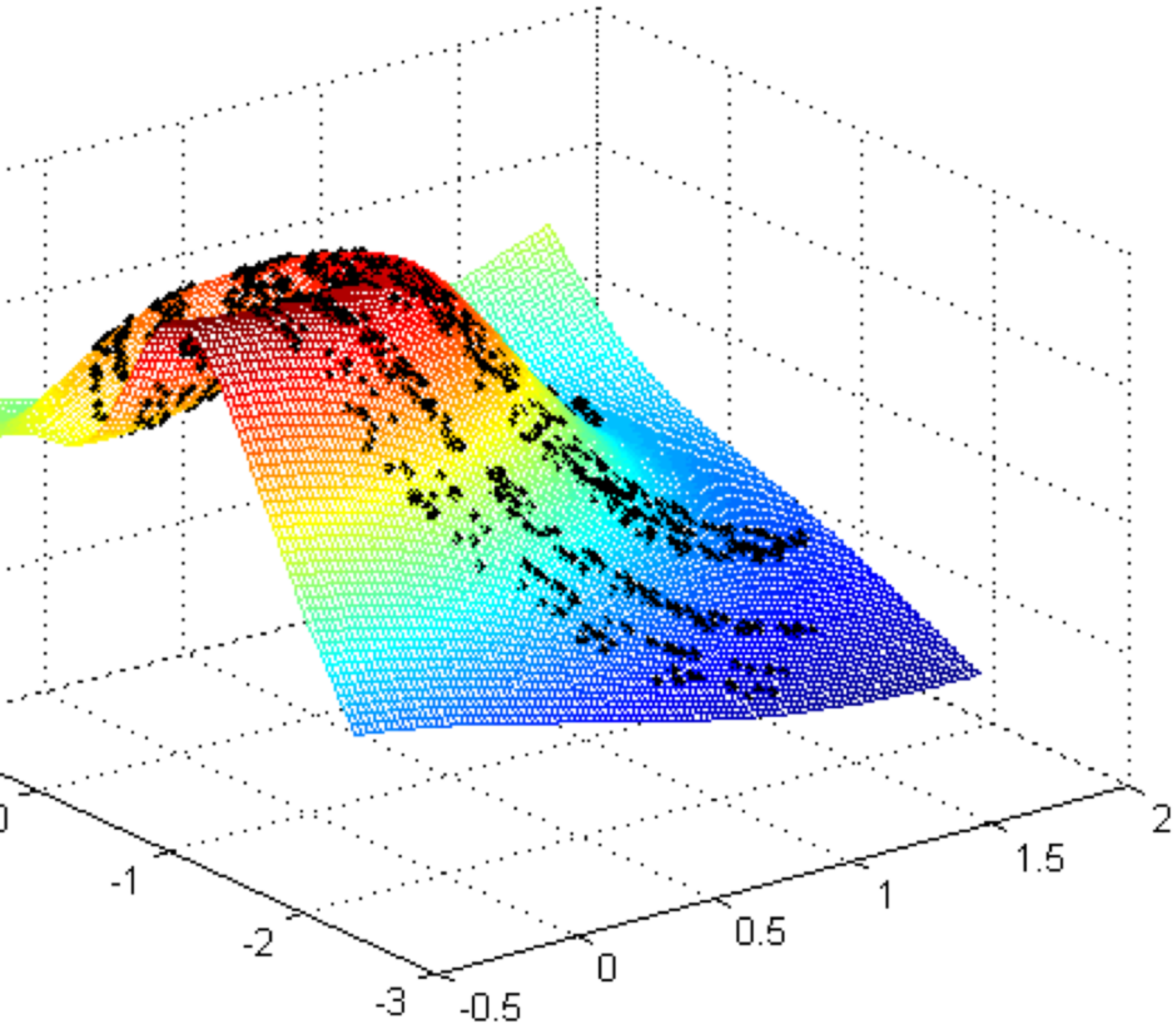

ikeda time series
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
-0.5
0
0.5
1
1.5
2

Density support

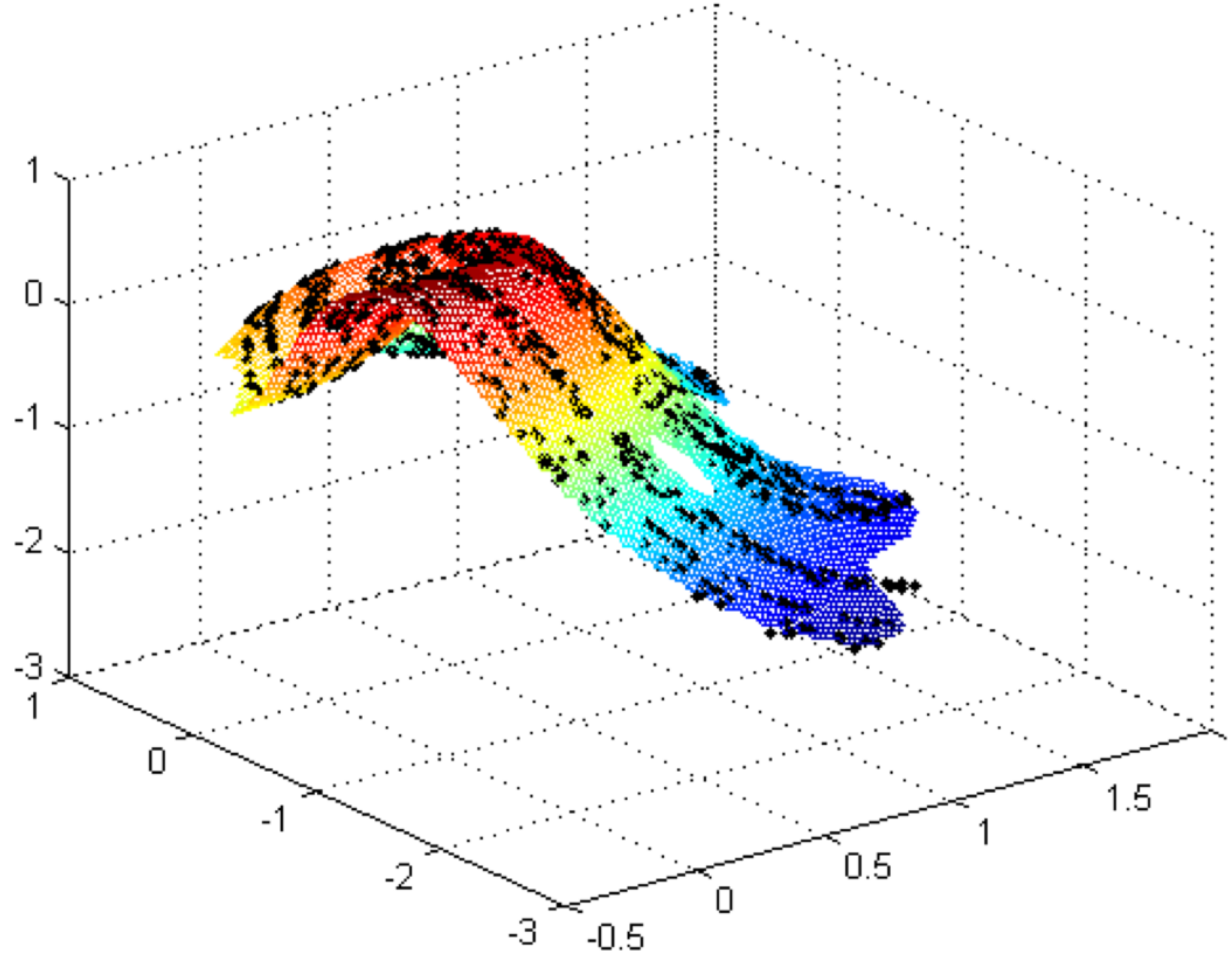


Support indictor function





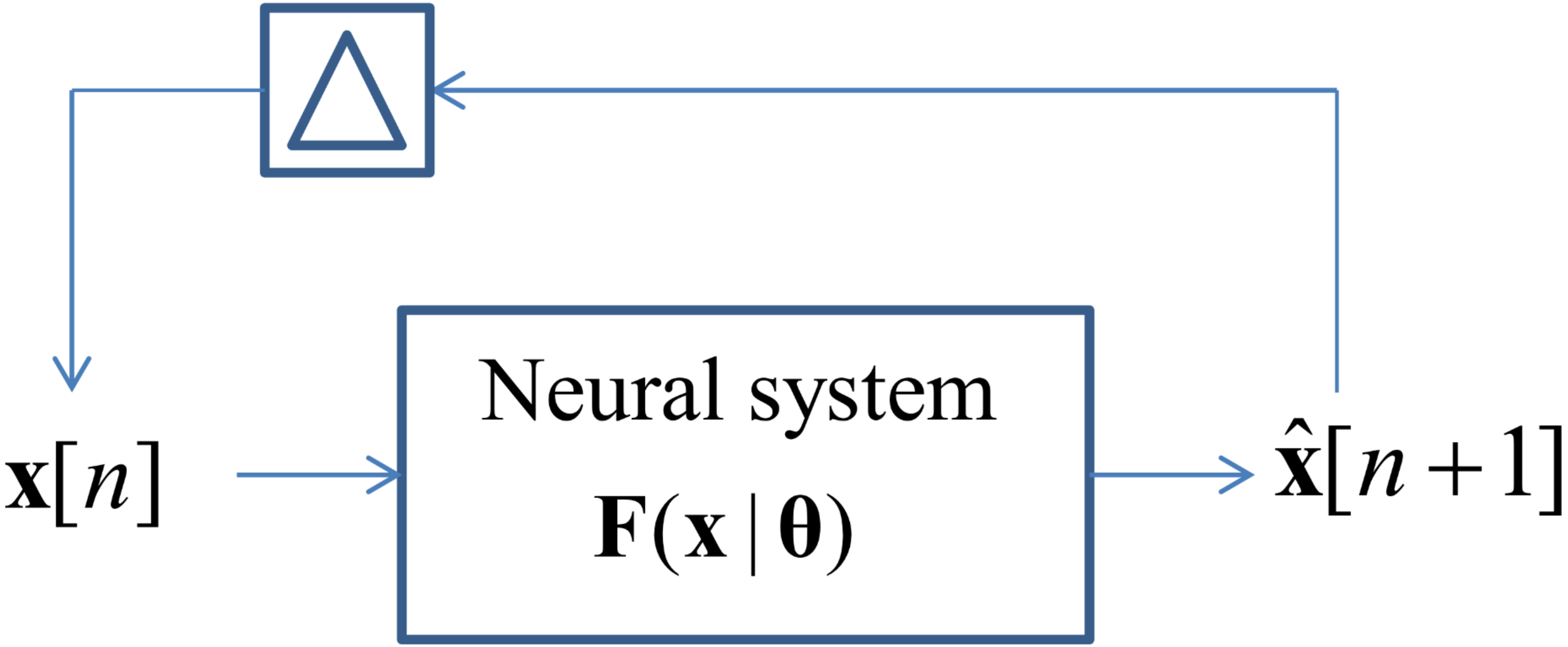
$f_2(x_1, x_2)$



$F_2(x_1, x_2 \mid \theta_2)$

MSE : Training error : 8.5348e-008

Testing error : 6.3944e-007



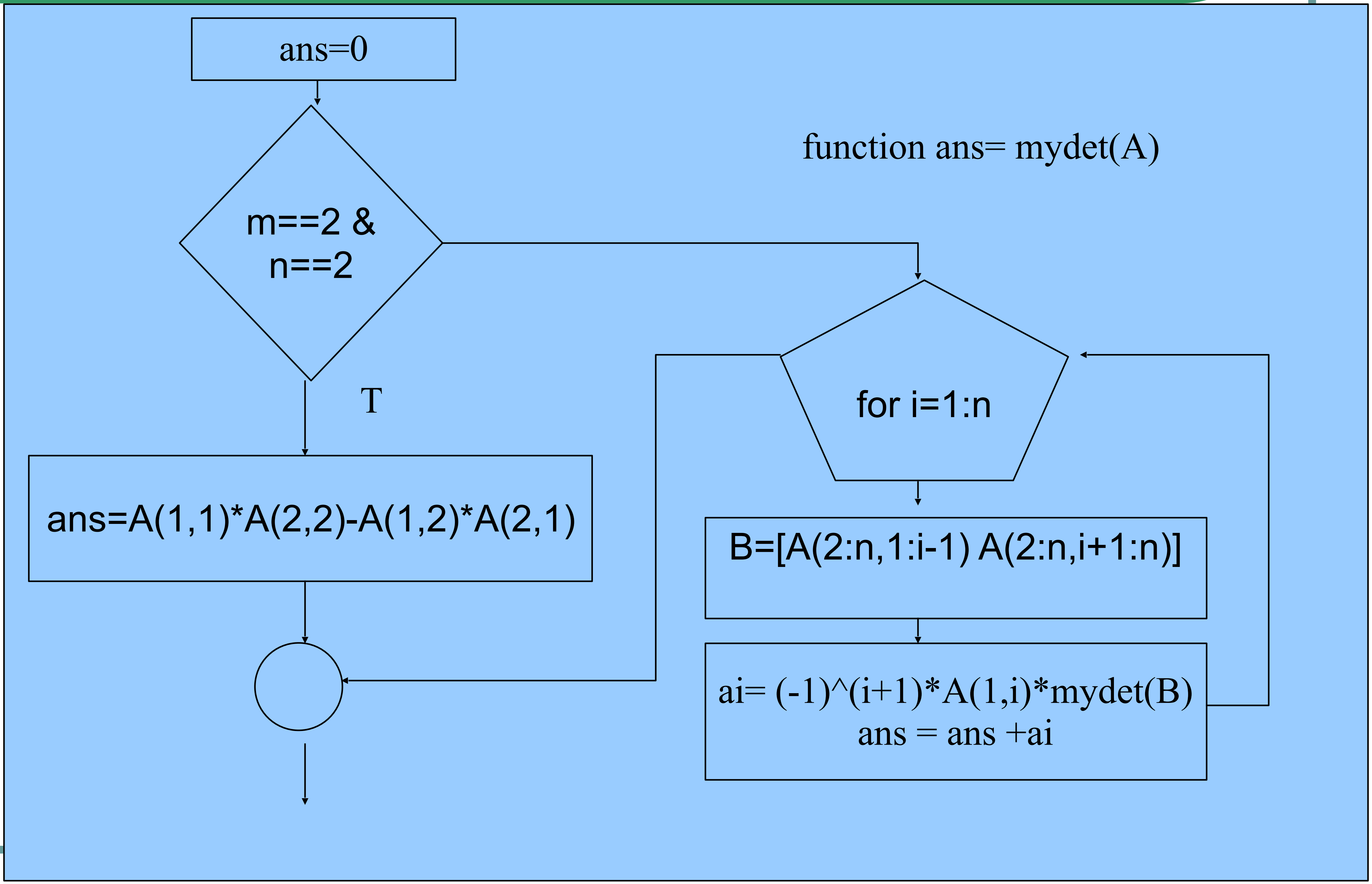
Δ
$\mathbf{x}[n]$
Neural system
$\mathbf{F}(\mathbf{x} \mid \boldsymbol{\theta})$
$\hat{\mathbf{x}}[n+1]$

Recurrent relation of Laplacian expansion

- det(A) is decomposed to n sub-tasks
- Each calculates determinant of an (n-1)-by-(n-1) matrix $\widetilde{A}_{1i}$
- The problem size is reduced from n to n-1

$$Det(A) = \sum_{i=1}^{n} (-1)^{i+1} a_{1i} \det(\widetilde{A}_{1i})$$

Recursive programming based on Laplacian expansion



Drawbacks

- Computational complexity, O(n!)
- Time consuming
- Memory consuming
- If n >10, it results in intolerant computing time to evaluate determinant by recursive programming.
- An improvement by Bareiss's standard fraction free Gaussian elimination

Bareiss's standard fraction free Gaussian elimination

Bareiss' standard fraction free Gaussian elimination (Bareiss, 1968).

$$A_{0,0}^{(-1)} = 1,$$

$$A_{i,j}^{(0)} = A_{i,j}, \text{ for } 1 \le i \le n, 1 \le j \le m,$$

$$A_{i,j}^{(k)} = \frac{A_{k,k}^{(k-1)} A_{i,j}^{(k-1)} - A_{i,k}^{(k-1)} A_{k,j}^{(k-1)}}{A_{k-1,k-1}^{(k-2)}}, \text{ for } 1 \le k < n, k < i, j \le m.$$

It is well known that

$$A_{i,j}^{(k)} = \begin{vmatrix} A_{1,1} & \cdots & A_{1,k} & A_{1,j} \\ \vdots & & \vdots & \vdots \\ A_{k,1} & \cdots & A_{k,k} & A_{k,j} \\ A_{i,1} & \cdots & A_{i,k} & A_{i,j} \end{vmatrix}.$$

Thus when $m = n$, $\det(A) = A_{n,n}^{(n-1)}$, and when $A = \begin{pmatrix} M & b \\ I & 0 \end{pmatrix}$, for square

```
[N,M]=size(A);
a=zeros(N,N,N);
```

for $1 \le k < n, k < i,j \le m$.

```
for k=1:N
for i=k+1:N
for j=k+1:N
```

$$A^{(-1)}_{0,0} = 1,$$

$$A^{(0)}_{i,j} = A_{i,j}, \text{ for } 1 \le i \le n, 1 \le j \le m,$$

$$A^{(k)}_{i,j} = \frac{A^{(k-1)}_{k,k} A^{(k-1)}_{i,j} - A^{(k-1)}_{i,k} A^{(k-1)}_{k,j}}{A^{(k-2)}_{k-1,k-1}},$$

```
for k=1:N
for i=k+1:N
for j=k+1:N
```

$$
\begin{aligned}
A_{0,0}^{(-1)} &= 1, \\
A_{i,j}^{(0)} &= A_{i,j}, \text{ for } 1 \le i \le n, 1 \le j \le m, \\
A_{i,j}^{(k)} &= \frac{A_{k,k}^{(k-1)} A_{i,j}^{(k-1)} - A_{i,k}^{(k-1)} A_{k,j}^{(k-1)}}{A_{k-1,k-1}^{(k-2)}},
\end{aligned}
$$

```
end
end
end
```

$$A_{0,0}^{(-1)} = 1,$$

$$A_{i,j}^{(0)} = A_{i,j}, \text{ for } 1 \leq i \leq n, 1 \leq j \leq m,$$

$$A_{i,j}^{(k)} = \frac{A_{k,k}^{(k-1)} A_{i,j}^{(k-1)} - A_{i,k}^{(k-1)} A_{k,j}^{(k-1)}}{A_{k-1,k-1}^{(k-2)}},$$

```
if k==1
a(i,j,k)=A(k,k)*A(i,j)-A(i,k)*A(k,j);
end

if k==2
a(i,j,k)=(a(k,k,1)* a(i,j,1)-a(i,k,1)* a(k,j,1))/A(k-1,k-1);
end

if k>2
a(i,j,k)=(a(k,k,k-1)* a(i,j,k-1)-a(i,k,k-1)* a(k,j,k-1))/a(k-1,k-1,k-2);
end
```

```
[N,M]=size(A);
a=zeros(N,N,N);
for k=1:N
for i=k+1:N
for j=k+1:N
    if k==1
    a(i,j,k)=A(k,k)*A(i,j)-A(i,k)*A(k,j);
    end
    if k==2
   a(i,j,k)=(a(k,k,1)* a(i,j,1)-a(i,k,1)* a(k,j,1))/A(k-1,k-1);
    end
    if k>2
    a(i,j,k)=(a(k,k,k-1)* a(i,j,k-1)-a(i,k,k-1)* a(k,j,k-1))/
a(k-1,k-1,k-2);
    end
end
end
end
a(N,N,N-1)
```

Bareiss' standard fraction free Gaussian elimination (Bareiss, 1968).

$$A_{0,0}^{(-1)} = 1,$$

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$$A_{i,j}^{(k)} = \frac{A_{k,k}^{(k-1)} A_{i,j}^{(k-1)} - A_{i,k}^{(k-1)} A_{k,j}^{(k-1)}}{A_{k-1,k-1}^{(k-2)}}, \text{ for } 1 \le k < n, k < i, j \le m.$$

It is well known that

$$A_{i,j}^{(k)} = \begin{vmatrix} A_{1,1} & \cdots & A_{1,k} & A_{1,j} \\ \vdots & & \vdots & \vdots \\ A_{k,1} & \cdots & A_{k,k} & A_{k,j} \\ A_{i,1} & \cdots & A_{i,k} & A_{i,j} \end{vmatrix}.$$

Thus when $m = n$, $\det(A) = A_{n,n}^{(n-1)}$, and when $A = \begin{pmatrix} M & b \\ I & 0 \end{pmatrix}$, for square