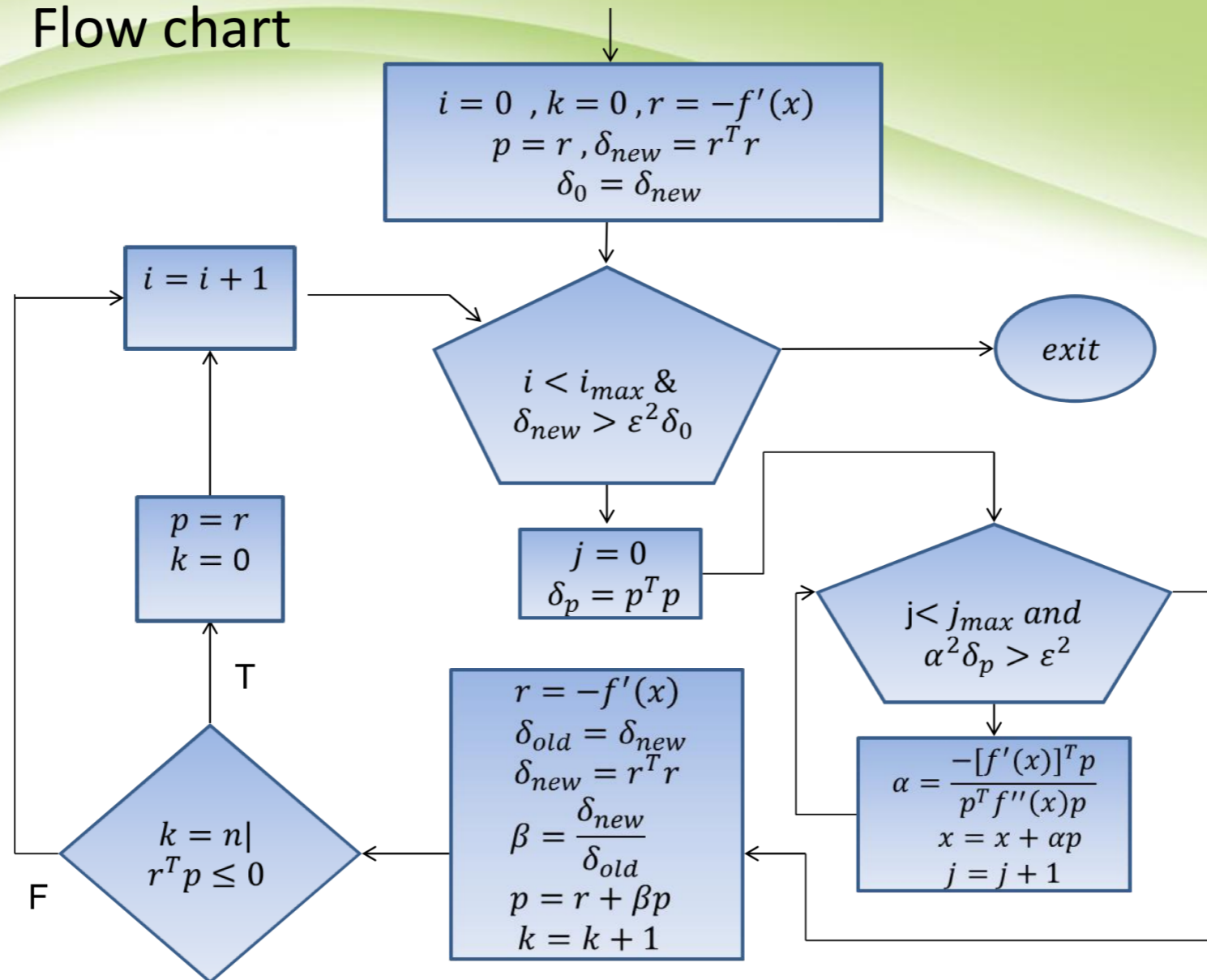


## Flow chart



# Implementation of Nonlinear CG method

# Expansion at $x = b$

Root at  $b$   
The second  
term is zero

$$f(x) \approx f(b) + f'(b)(x-b) + \frac{f''(b)}{2}(x-b)^2$$

# Root at $x=b$

The third term is negligible for some positive small  $\varepsilon$  if

$$\frac{1}{2} f''(b)(x-b)^2 < \varepsilon \cdot f(b)$$
$$\text{or } (x-b)^2 < 2\varepsilon \frac{f(b)}{f''(b)}$$
$$\frac{|x-b|}{b} < \sqrt{2\varepsilon} \sqrt{\frac{|f(b)|}{b^2 f''(b)}}$$

# Halting condition

The reason for writing the inequality in this way is that the left hand side is the relative error in  $x$ , and that for most functions, the final square root is a number of order unity.

Therefore, as a rule of thumb, it is hopeless to ask for bracketing with a width of less than  $\sqrt{\epsilon}$ . Knowing this inescapable fact will save you a lot of useless bisection!

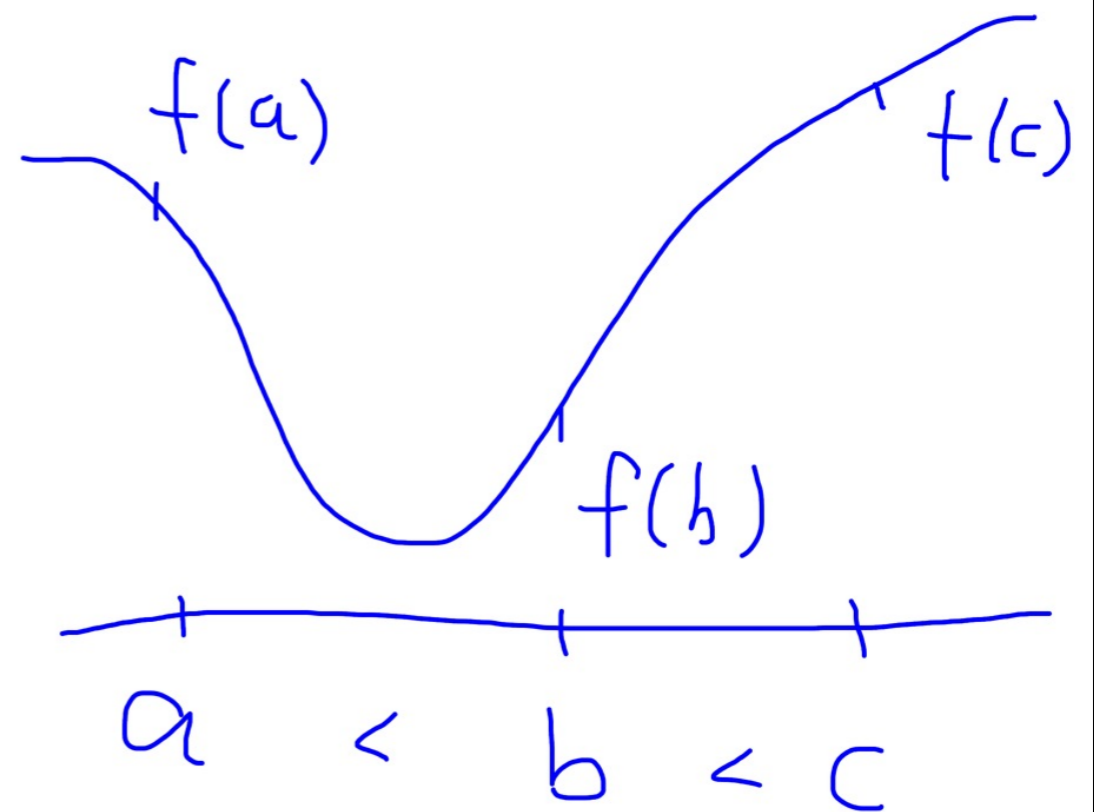
# Choose a new x

- a, b and c define a bracket

$$\frac{b-a}{c-a} = W$$

$$\frac{c-b}{c-a} = 1 - W$$

$$\frac{x-b}{c-a} = Z$$

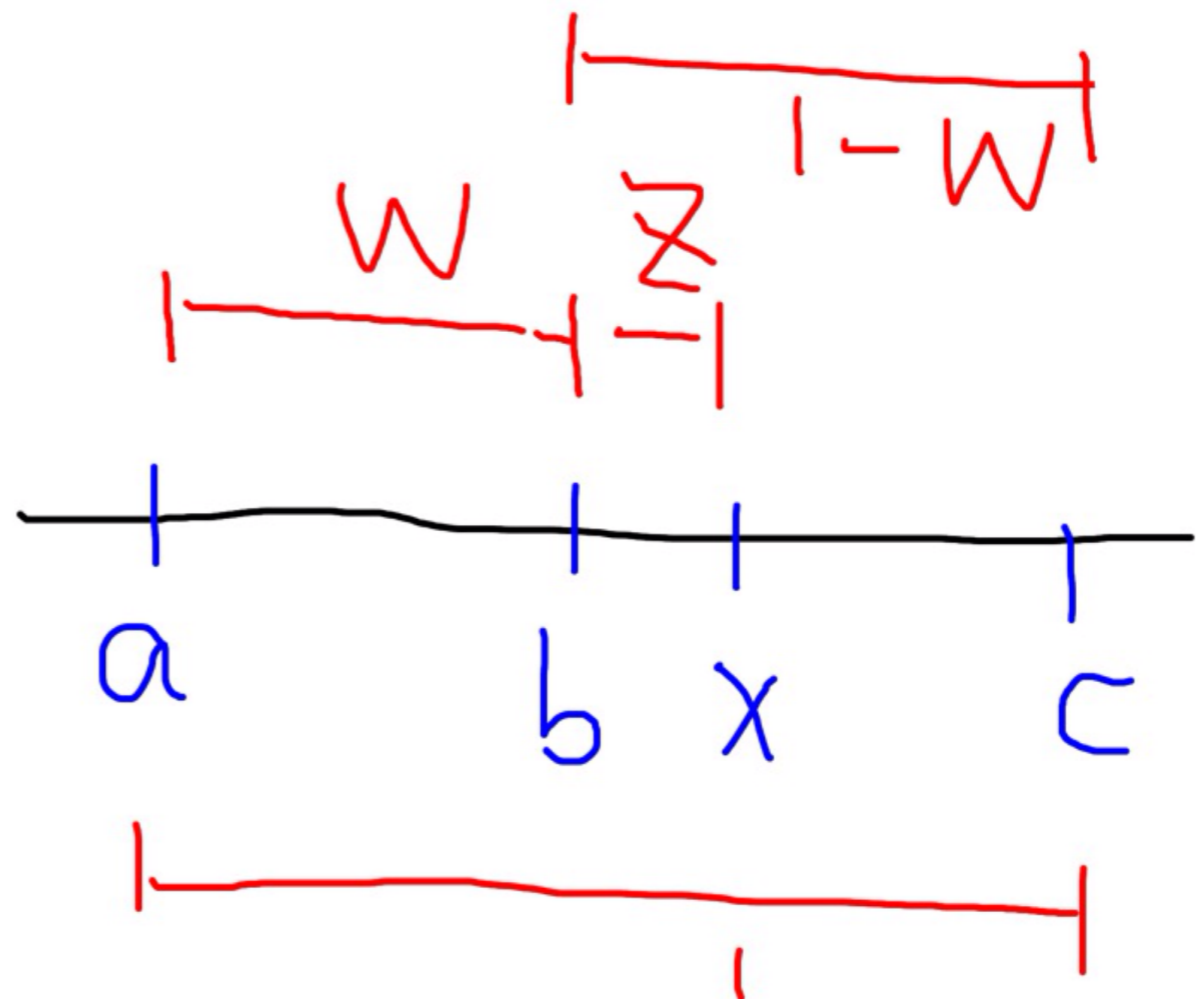


# Next two intervals

- The length of the next bracket will be  $W+Z$  or  $1-W$

$$W + Z = 1 - W$$

$$Z = 1 - 2W$$

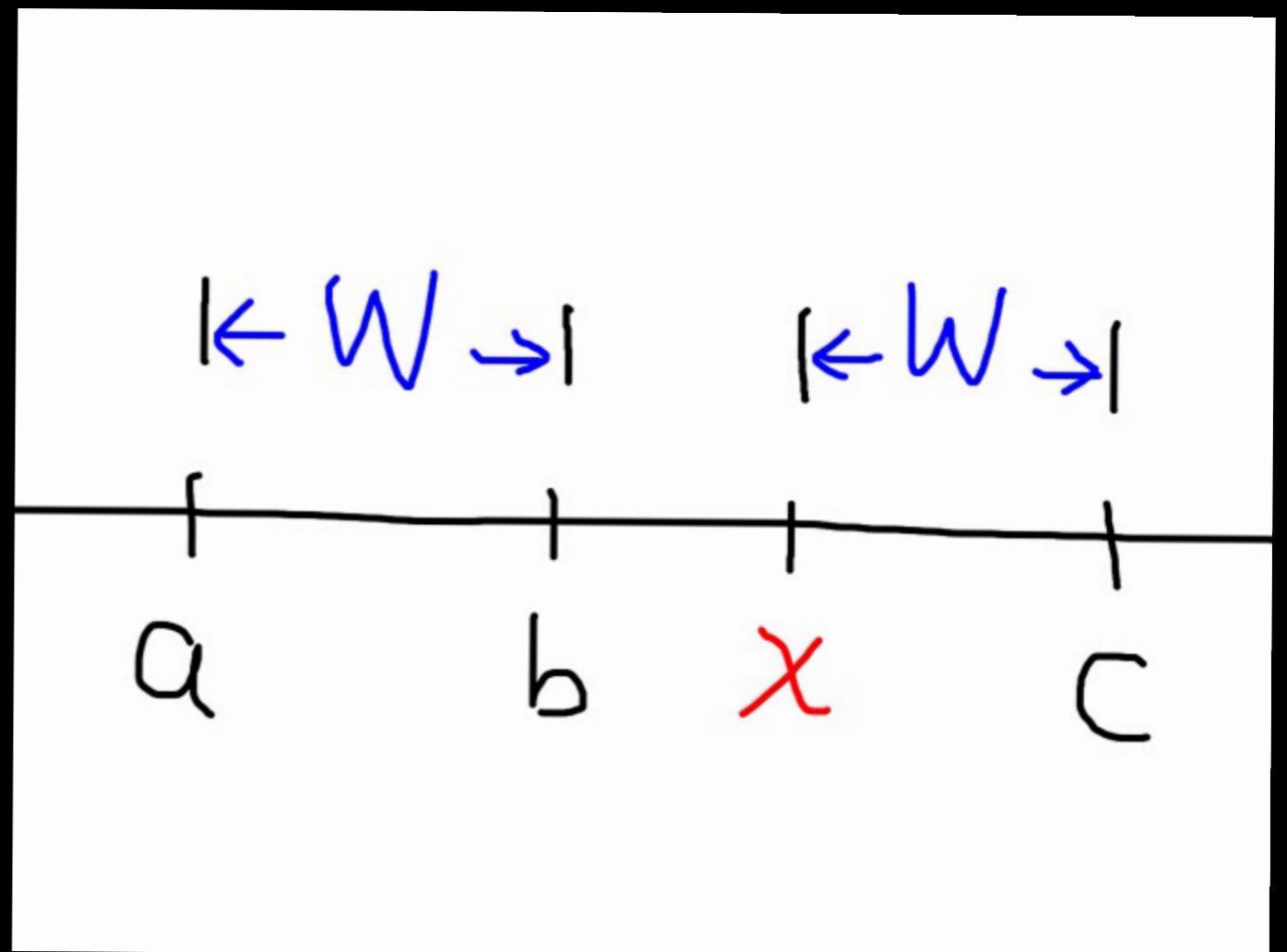


# Choose a new $x$

the new point is the symmetric point to  $b$  in the interval, namely with  $|b-a|$  equal to  $|c-x|$ .

$$W < 1/2$$

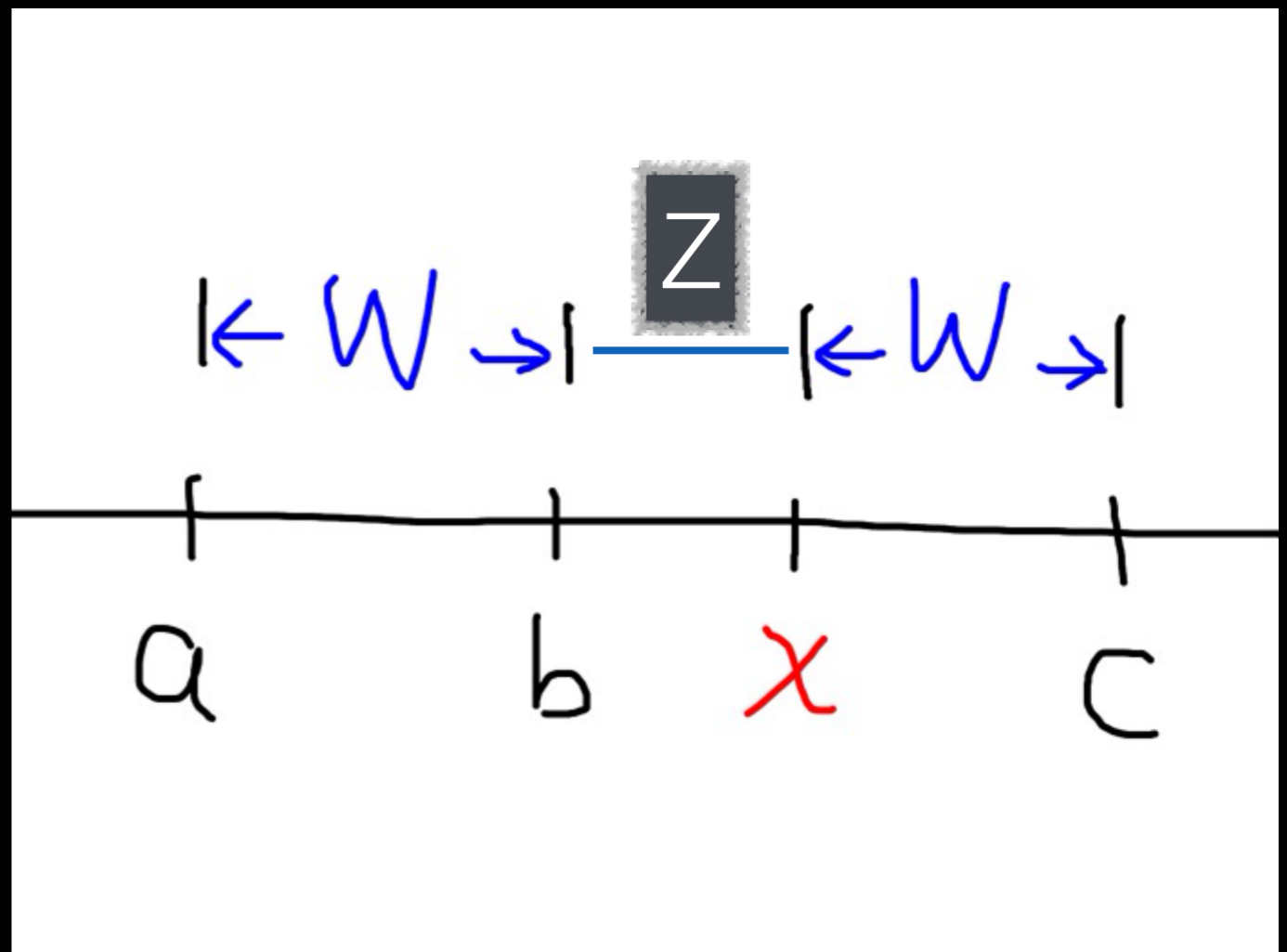
$x$  lies on the larger segment



# To maintain a constant ratio

- The next bracket will be in length of  $1-W$
- $a, b, x$  or  $b, x, c$

$$\frac{Z}{1-W} = \frac{W}{1}$$





# Why golden ratio?

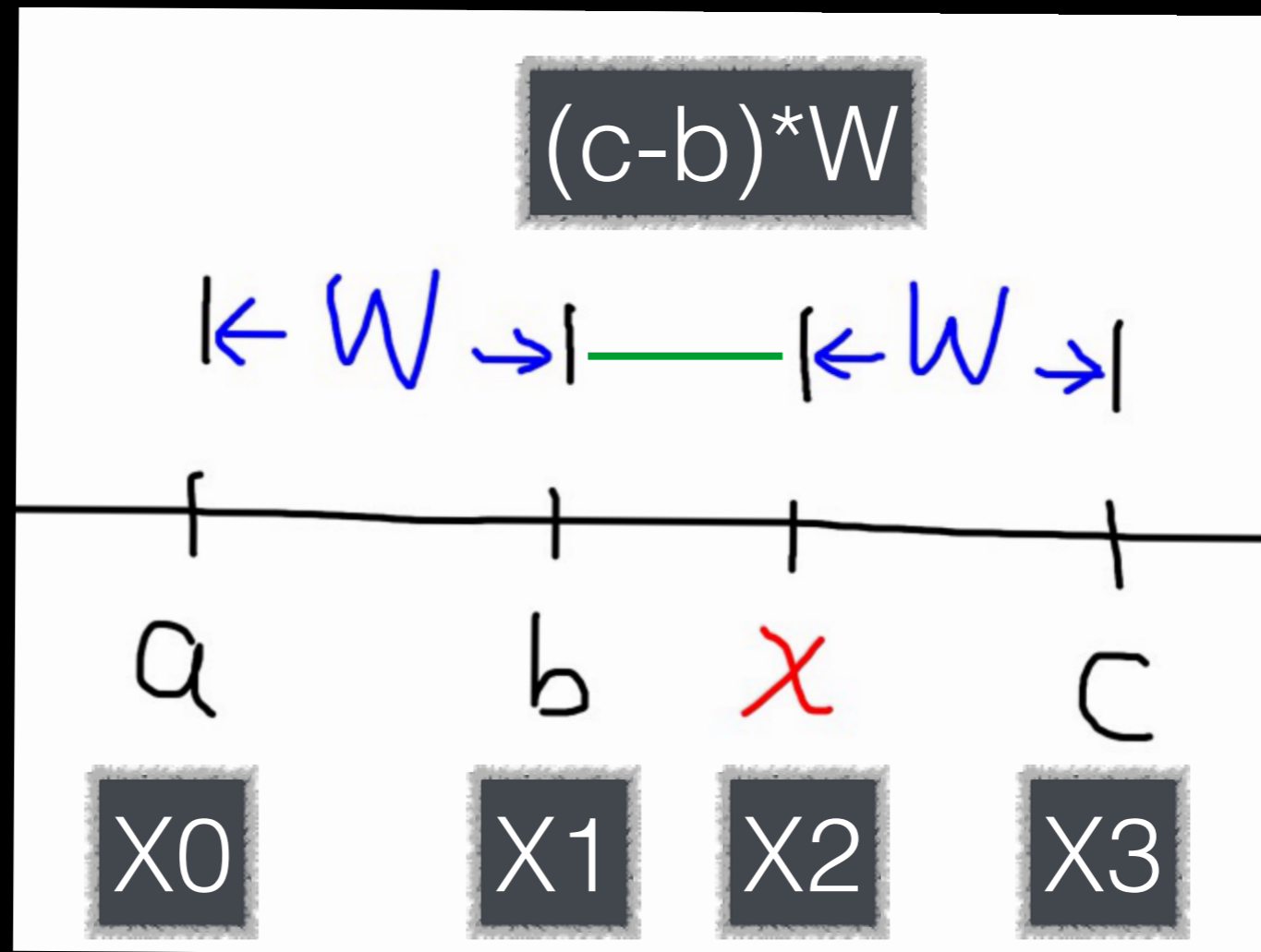
$$Z = 1 - 2W \quad (1)$$

$$\frac{Z}{1-W} = \frac{W}{1} \quad (2)$$

$$W^2 - 3W + 1 = 0$$

$$W = \frac{3 - \sqrt{5}}{2}$$

# Setting



function Brent (a,b,c,f)

$C = (3 - \sqrt{5}) / 2$   
 $x_0 = a; x_3 = c;$

$\text{abs}(c-b) > \text{abs}(a-b)$

$x_1 = b;$   
 $x_2 = b + C * (c - b)$

$x_2 = b;$   
 $x_1 = b - C * (b - a)$

$f_1 = f(x_1); f_2 = f(x_2)$

Not  
Halting  
cond

if  $f_1 < f_2$   
 $x_{\min} = x_1;$   
 $f_{\min} = f_1$   
else  
 $x_{\min} = x_2;$   
 $f_{\min} = f_2$   
end

EXIT

$f_2 < f_1$

$x_0 = x_1; x_1 = x_2;$   
 $x_2 = x_1 + C * (x_3 - x_1)$

$x_3 = x_2; x_2 = x_1;$   
 $x_1 = x_2 + C * (x_0 - x_2)$

$f_1 = f(x_1); f_2 = f(x_2)$

# Halting condition

$$\begin{aligned} & \text{abs}(x3-x0) > \\ & \text{tol}(\text{abs}(x1)+\text{abs}(x2)) \end{aligned}$$

# Parabolic interpolation

- A quadratic polynomial that passes  $(a, f(a))$ ,  $(b, f(b))$  and  $(c, f(c))$
- Find the minimum

# How to select x

iPad

下午10:59

22%

$(a \ f_a), (b \ f_b), (c \ f_c)$

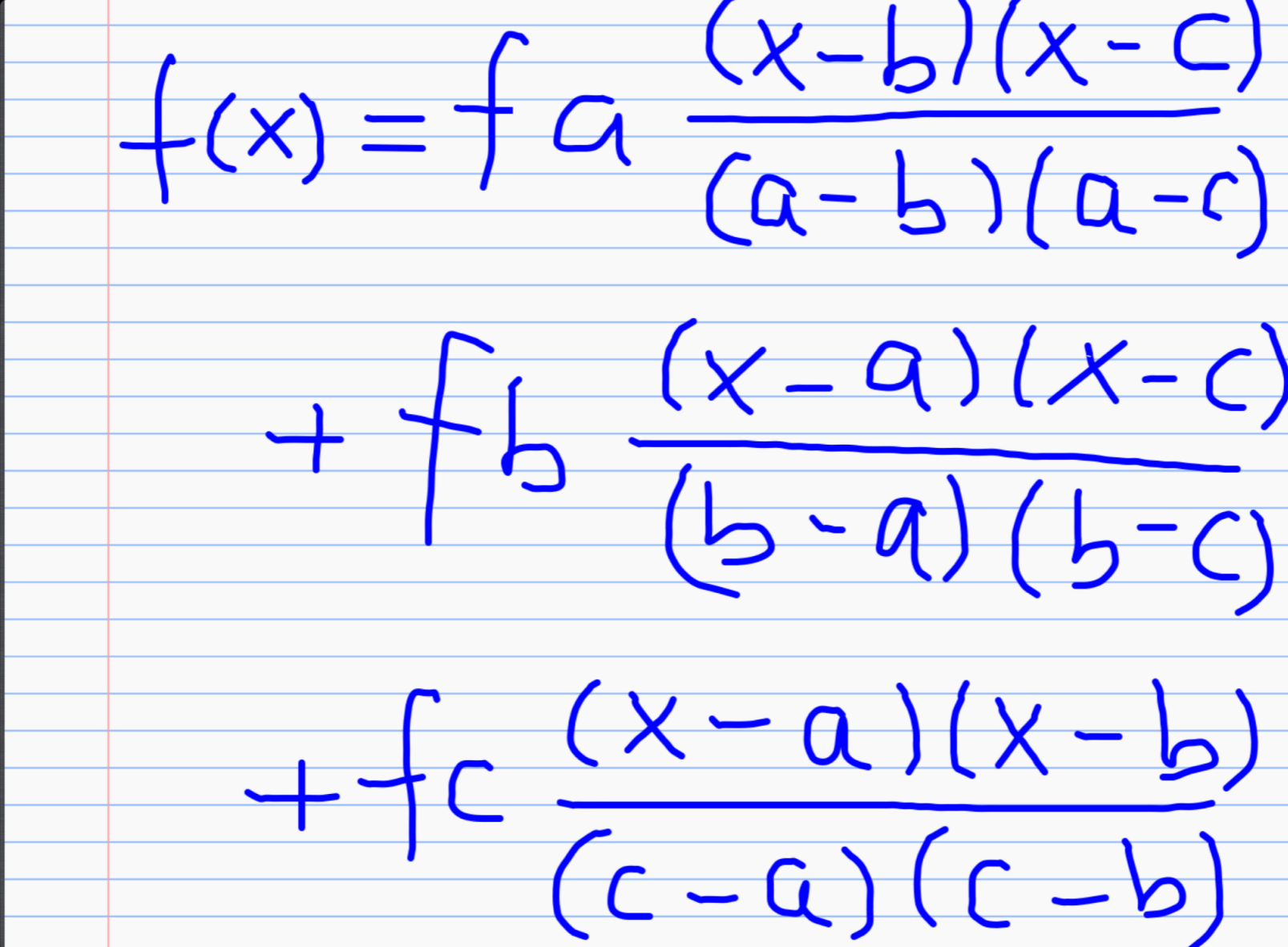
$$x = b^{-\frac{1}{2}} \frac{(b-a)^2 [f_b - f_c]}{(b-a)(f_b - f_c)}$$

$$\frac{-(b-c)^2 [f_b - f_a]}{-(b-c)[f_b - f_a]}$$

$$-(b-c)[f_b - f_a]$$

# Quadratic polynomial

A linear combination of three Lagrange polynomials



The image shows a handwritten mathematical formula on a notepad. The formula represents a quadratic polynomial as a linear combination of three Lagrange basis polynomials. The notepad has a white background with blue horizontal lines. At the top, there is a status bar with 'iPad', a signal strength icon, the time '下午10:59', and a battery icon showing '22%'.

$$f(x) = f_a \frac{(x-b)(x-c)}{(a-b)(a-c)} + f_b \frac{(x-a)(x-c)}{(b-a)(b-c)} + f_c \frac{(x-a)(x-b)}{(c-a)(c-b)}$$

# Find the minimum

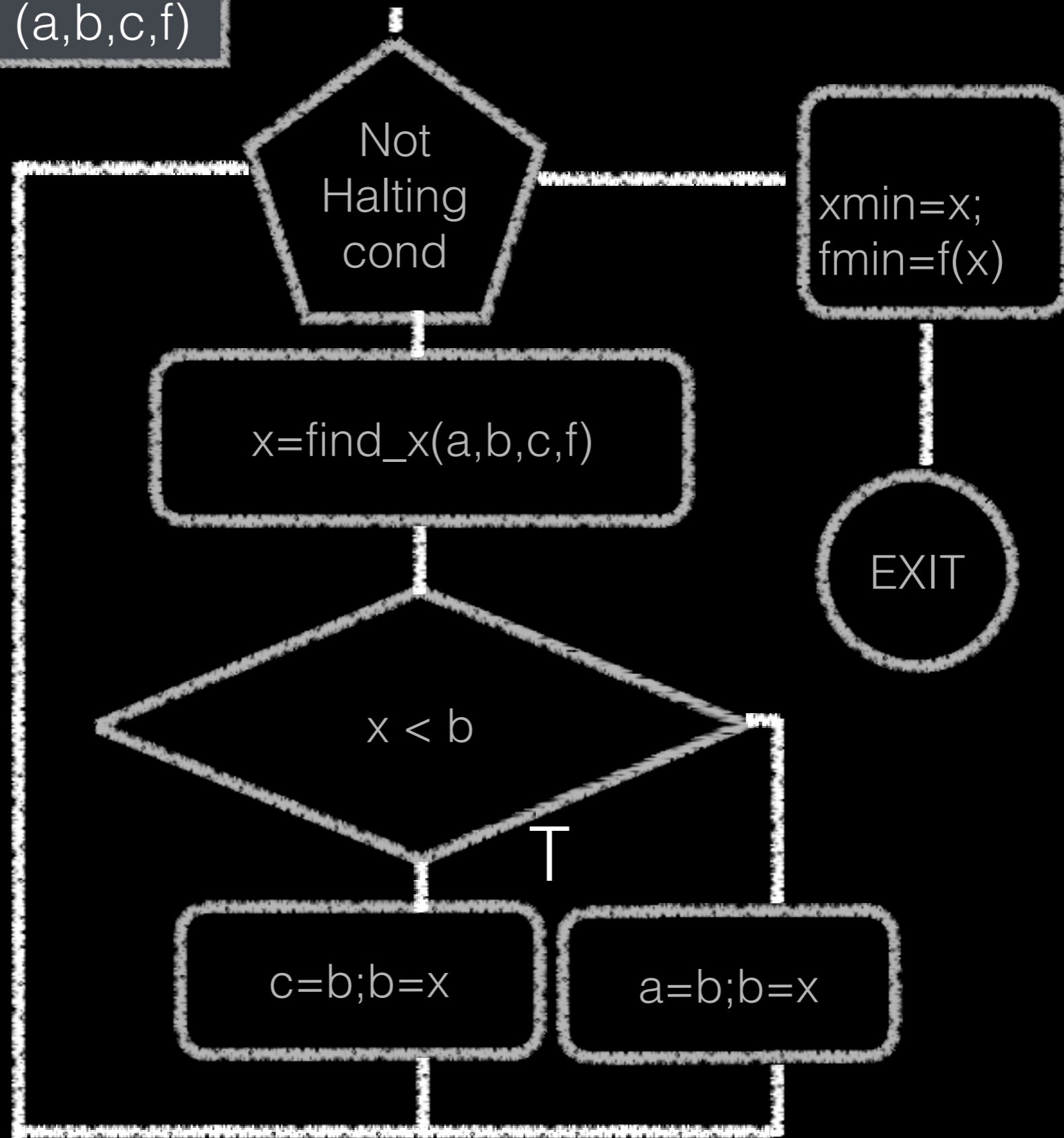
- Set zero to the interpolating quadratic polynomial

The image shows a digital notepad with a white background and blue horizontal lines. At the top, there is a status bar with 'iPad', a Wi-Fi icon, the time '下午10:59', and a battery icon showing '22%'. The main content is a handwritten equation in blue ink, representing the Lagrange interpolation formula for a quadratic polynomial. The equation is written in three lines, with the final result set to zero. The first line is  $f(x) = f_a \frac{(x-b)(x-c)}{(a-b)(a-c)}$ . The second line is  $+ f_b \frac{(x-c)(x-a)}{(b-c)(b-a)}$ . The third line is  $+ f_c \frac{(x-a)(x-b)}{(c-a)(c-b)} = 0$ .

$$f(x) = f_a \frac{(x-b)(x-c)}{(a-b)(a-c)} + f_b \frac{(x-c)(x-a)}{(b-c)(b-a)} + f_c \frac{(x-a)(x-b)}{(c-a)(c-b)} = 0$$



function parabolic (a,b,c,f)



# mnbrak.m

mnbrak.m

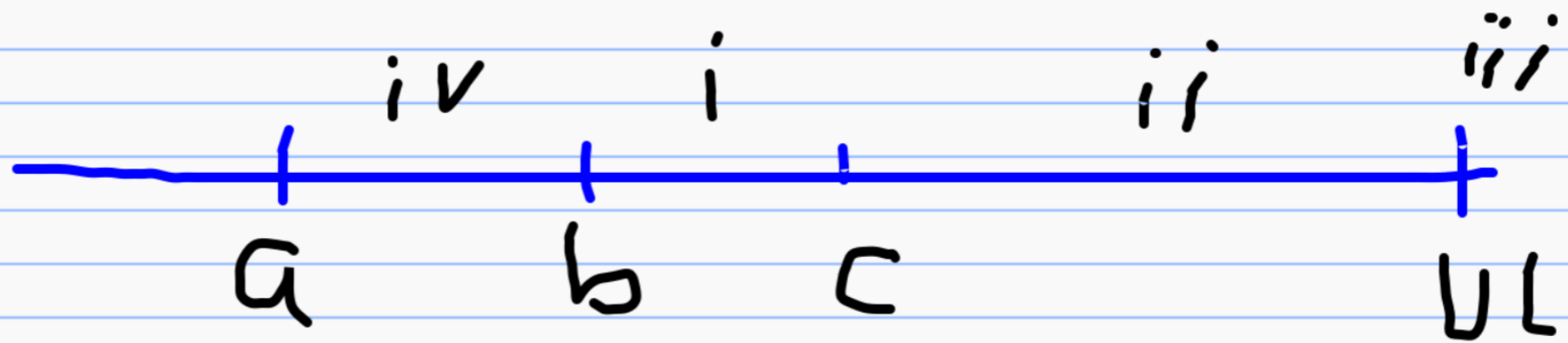
linmin.m

linmin.m

# Routine for Initially Bracketing a Minimum

- A minimum is known to be bracketed only when there is a triplet of points,  $a < b < c$ , such that  $f(b)$  is less than both  $f(a)$  and  $f(c)$ .

$$a < b, f a > f b$$



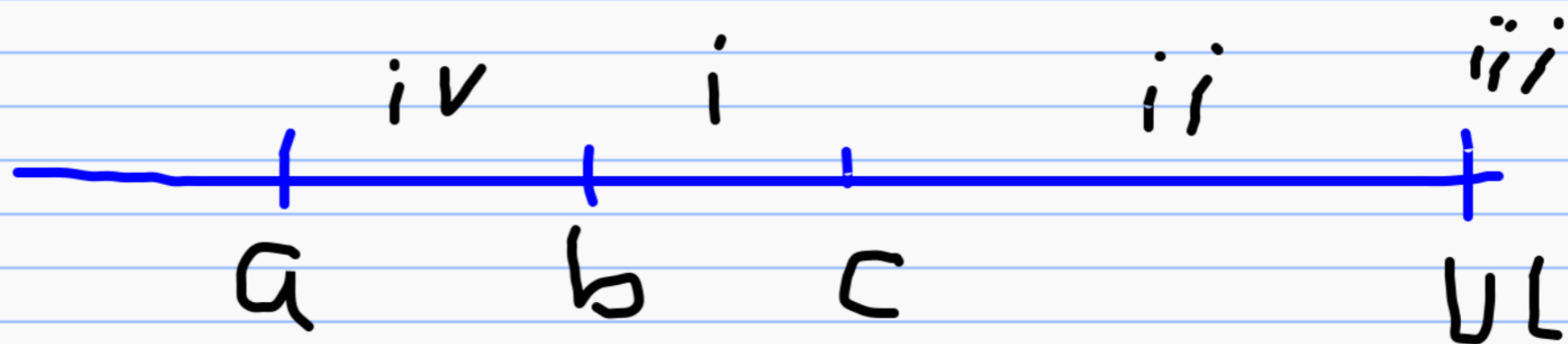
$$c \leftarrow b + G(b - a)$$


$$uL \leftarrow b + 1000(c - b)$$

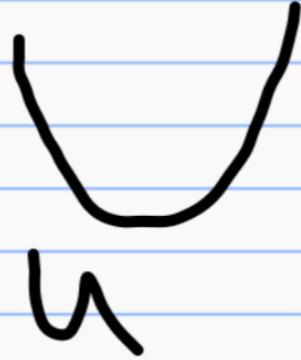
$$u \leftarrow \min_g(a, b, c)$$

(i)  $\bigvee$   
 $u$   $(a, u, c)$

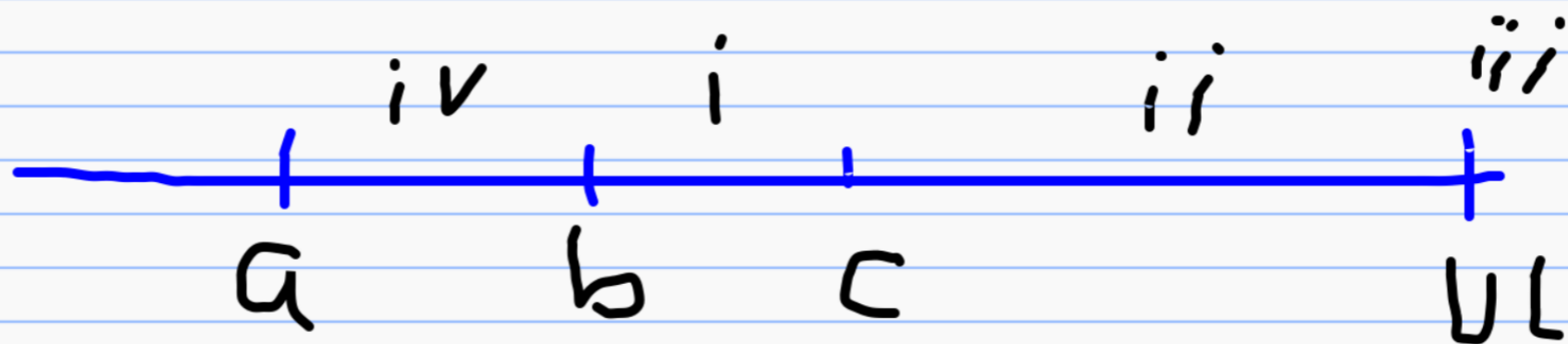
$$a < b, f a > f b$$



(i)   $(a, b, u)$

(ii)   $v \leftarrow u + G(u - c)$   
 $(c, u, v)$

$$a < b, f a > f b$$



$$(iii) u \leftarrow uL$$

$$(b, c, u)$$

$$(iv) u \leftarrow c + \frac{1}{5}(c-b)$$

$$(b, c, u)$$

# Case i

Two situations in case i return a bracket  
The other situations need further iterations for bracketing

# Routines

- Switch roles of a and b such that  $f_a > f_b$
- First guess for c.
- While not HC
  - Compute u by parabolic extrapolation from a, b, c.  
TINY is used to prevent any possible division by zero.
  - u in ( b, c)
    - $f_c > f_u$  : return (b,u,c)
    - $f_b < f_u$  : return (a,b,u)
  - u in (c, uL) :  $v \leftarrow c + G(u-c)$ ; (a,b,c)  $\leftarrow$  (c,u,v)
  - Parabolic fit is between c and its limit.
  - $u > uL$ :  $u \leftarrow uL$ ; (a,b,c)  $\leftarrow$  (b,c,u)
    - %Reject parabolic u, use default magnification.
    - %Limit parabolic u to maximum allowed value.
  - $u < b$ 
    - Eliminate oldest point and continue.



# mnbrak.m

mnbrak.m

[http://meteo.macc.unican.es/prometeo/temp/carmen/07\\_NNMeteo/dani/viento\\_ordenado/toolboxes/adaboost/@rbf\\_net\\_w/private/mnbrak.m](http://meteo.macc.unican.es/prometeo/temp/carmen/07_NNMeteo/dani/viento_ordenado/toolboxes/adaboost/@rbf_net_w/private/mnbrak.m)

linmin.m

[http://meteo.macc.unican.es/prometeo/temp/carmen/07\\_NNMeteo/dani/viento\\_ordenado/toolboxes/adaboost/@rbf\\_net\\_w/private/linmin.m](http://meteo.macc.unican.es/prometeo/temp/carmen/07_NNMeteo/dani/viento_ordenado/toolboxes/adaboost/@rbf_net_w/private/linmin.m)

linmin.m

<http://www.wiley.com/legacy/wileychi/koopbayesian/supp/linmin.m>