

Conjugate vectors

$$P_{k+1} = r_{k+1} - \sum_{i \leq k} \frac{\langle P_i, r_{k+1} \rangle_A}{\langle P_i, P_i \rangle_A} P_i$$

$$P_j^T A P_{k+1} = P_j^T A r_{k+1} -$$

$$P_j^T A \frac{\langle P_j, r_{k+1} \rangle_A}{\langle P_j, P_j \rangle_A} P_j = 0$$

$$\forall j \leq k$$

Flow chart

Guess x_0

$r = b - A * x_0$; $p = r$

$k = 0$; $x = x_0$

Repeat

$a = r' * r / p' * A * p$

$x = x + a * p$

$r_{\text{new}} = r - a * A * p$

if r_{new} is sufficiently small exit

$b = r_{\text{new}}' * r_{\text{new}} / r' * r$

$p = r_{\text{new}} + b * p$

$k = k + 1$; $r = r_{\text{new}}$;

end repeat

Guess x_0

$r = b - A * x_0$; $p = r$; $k = 0$; $x = x_0$

$\text{norm}(r) < \text{epsilon}$

$a = r^T r / p^T A p$

$x = x + a * p$

$r_{\text{new}} = r - a * A * p$

$b = r_{\text{new}}^T r_{\text{new}} / r^T r$

$p = r_{\text{new}} + b * p$

$k = k + 1$; $r = r_{\text{new}}$;

END

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A=[1 2; 2 0];b=[1 1]';x0=rand(1,2)';  
r=b-A*x0; p=r;  
k=0;x=x0;ep=10^-6;  
while ~(norm(r) < ep)
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    a=r'*r/(p'*A*p);
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    x=x+a*p;
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    r_new=r-a*A*p;
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    b=r_new'*r_new/(r'*r);
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```
    p=r_new+b*p;
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```
    k=k+1; r=r_new;
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end
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Prove conjugate