Independent component analysis

- Statistical independency between random variables

- fundamental probability theory
- linear transformation
- Kullback Leibler divergence
- Marginal density estimation

- multichannel observations
- Markov process
- multichannel time series analysis
- linear mixture assumption
- convolutive mixture assumption
- blind source separation

- problem statement
- algorithms
  - Fast ICA: http://research.ics.tkk.fi/ica/fastica/
  - Jade ICA: http://perso.telecomparistech.fr/~cardoso/stuff.html
  - Potts ICA

- applications
  - Fetal ECG extraction
  - EEG analysis
  - functional MRI analysis

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1. Title: **Annealed Kullback-Leibler divergence minimization for generalized TSP, spot identification and gene sorting**
Author(s): Wu Jiann-Ming; Hsu Pei-Hsun
Source: NEUROCOMPUTING Volume: **74** Issue: **12-13** Pages: **2228-2240** DOI: **10.1016/j.neucom.2011.03.002** Published: **JUN 2011**
Times Cited: **0** (from Web of Science)
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2. Title: **Independent component analysis based on marginal density estimation using weighted Parzen windows**
Author(s): Wu Jiann-Ming; Chen Meng-Hong; Lin Zheng-Han
Source: NEURAL NETWORKS Volume: **21** Issue: **7** Pages: **914-924** DOI: **10.1016/j.neunet.2008.01.005** Published: **SEP 2008**
Times Cited: **0** (from Web of Science)
[View abstract]

3. Title: **Blind separation of fetal electrocardiograms by annealed expectation maximization**
Author(s): Wu Jiann-Ming
Source: NEUROCOMPUTING Volume: **71** Issue: **7-9** Pages: **1500-1514** DOI: **10.1016/j.neucom.2007.05.009** Published: **MAR 2008**
Times Cited: **2** (from Web of Science)
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4. Title: **Learning generative models of natural images**
Author(s): Wu JM; Lin ZH
Source: NEURAL NETWORKS Volume: **15** Issue: **3** Pages: **337-347** Article Number: **PII S0893-6080(02)00018-7** DOI: **10.1016/S0893-6080(02)00018-7** Published: **APR 2002**
Times Cited: **5** (from Web of Science)
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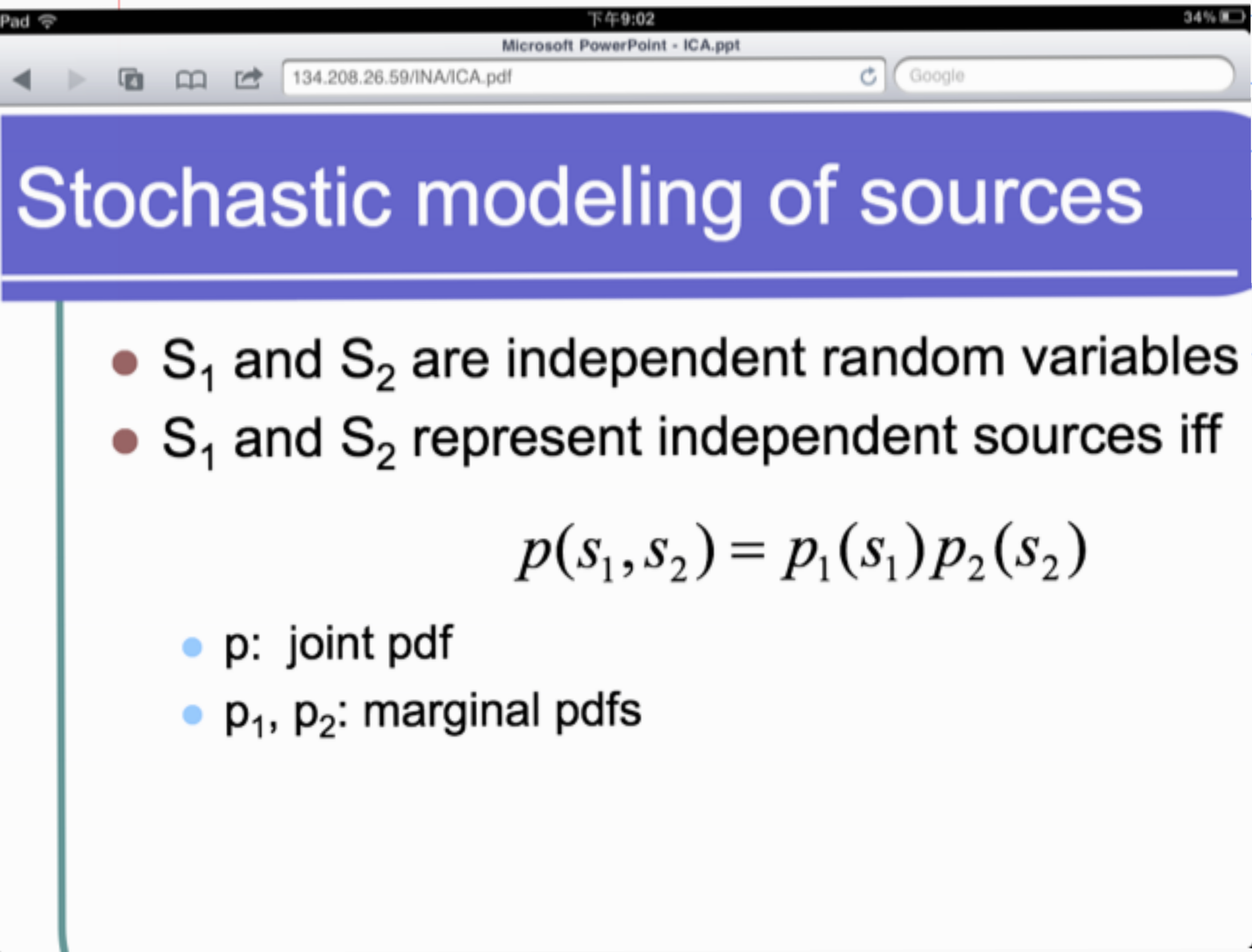
Statistical independency of two random variables



Linear transformation of two random variables

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = A \begin{bmatrix} s_1 \\ s_2 \end{bmatrix}$$

- S1 and S2 are independent
- x1 and x2 may not independent

$A_{2\times 2}$ $\quad A \sim I$

{ S1(t) } sampled from S1
{ S2(t) } sampled from S2

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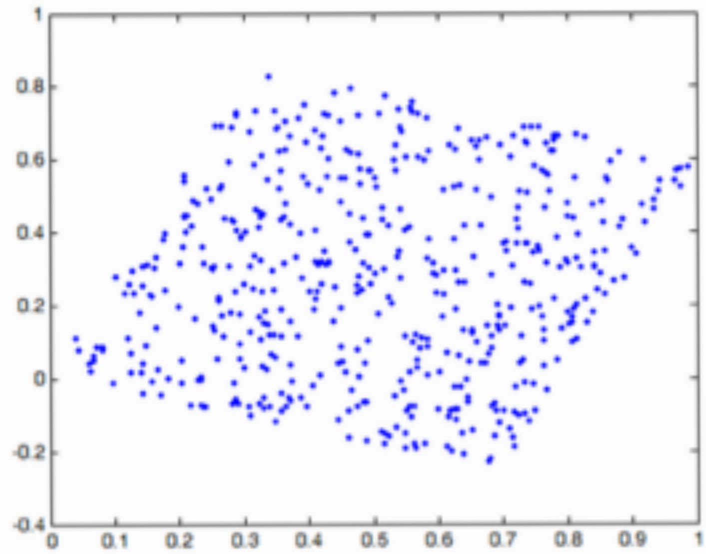
- Instant mixing

$$\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \mathbf{A} \begin{bmatrix} s_1(t) \\ s_2(t) \end{bmatrix}$$

```
A=[0.7 0.31;-0.25 0.85];
x=A*s;
plot(x(1,:),x(2,:),'.');
```

```
s=rand(2,800);
A=[0.7 0.31;-0.25 0.85];
x=A*s;
plot(x(1,:),x(2,:),'.');
```

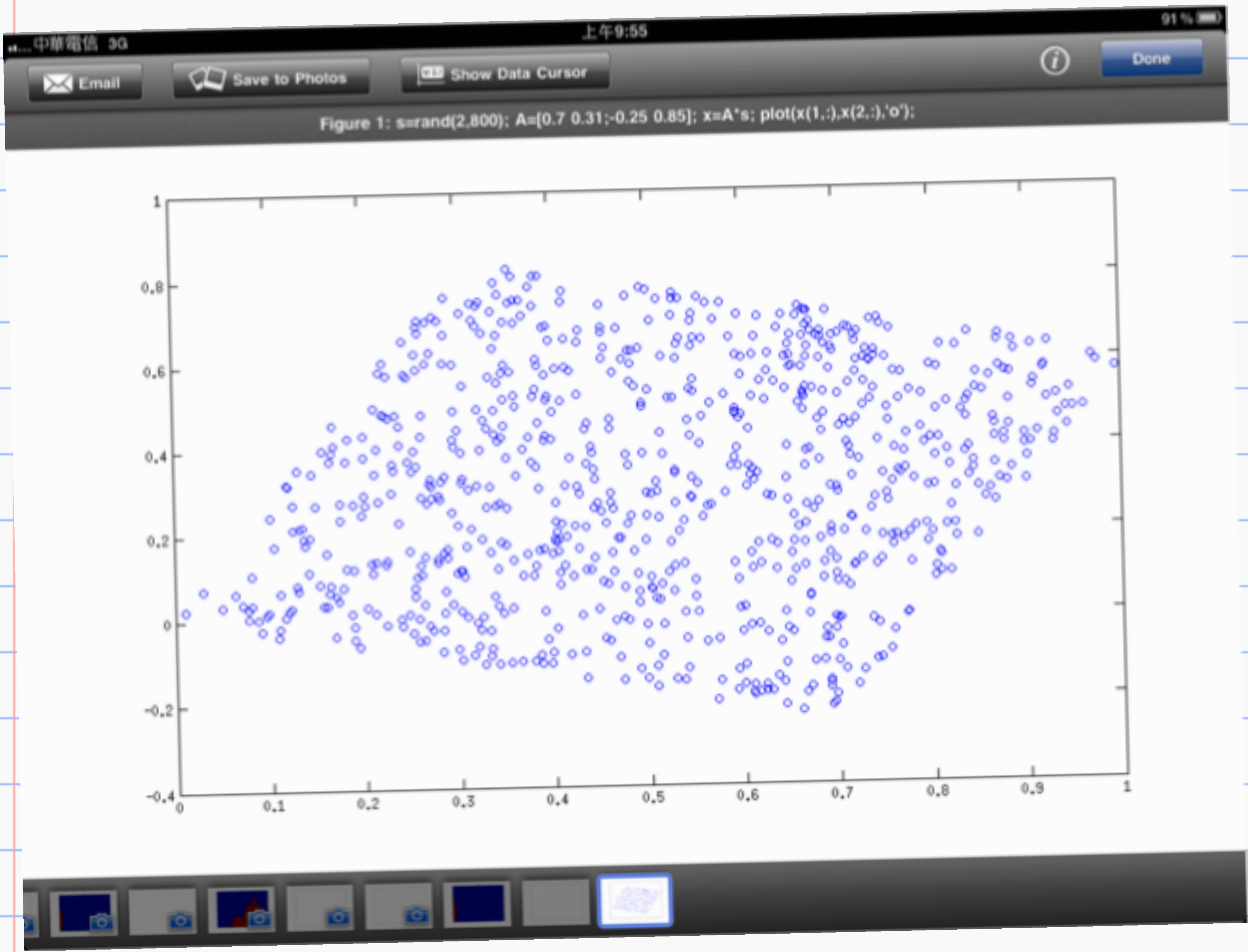
x2



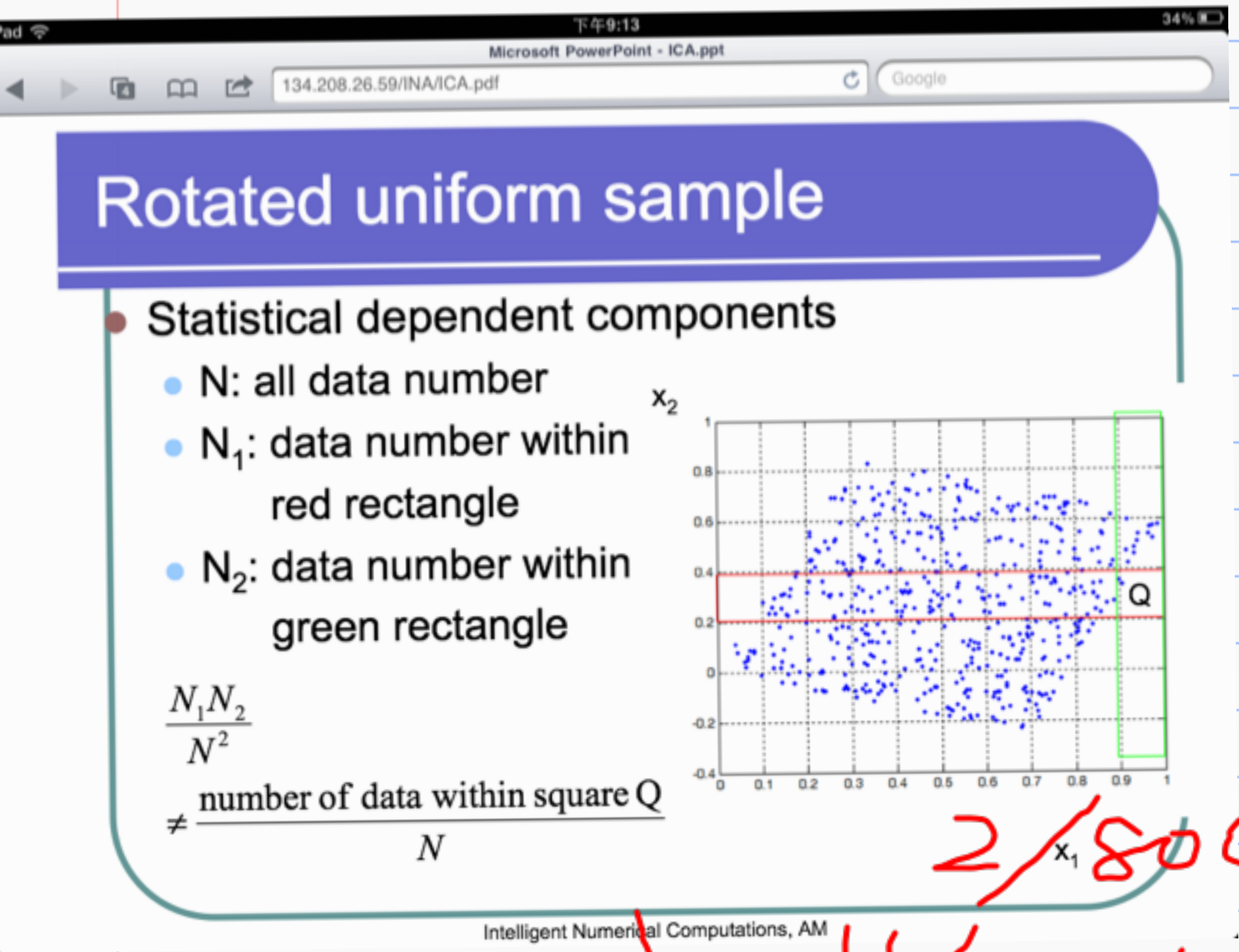
x1

Intelligent Numerical Computations, AM
NDHU

```
s=rand(2,800);
A=[0.7 0.31;-0.25 0.85];
x=A*s;
plot(x(1,:),x(2,:),'o');
```



Statistical dependency of linear mixtures of two independent random numbers



$$2/800 \neq 11/800 \times \frac{80}{800}$$

How to quantify statistical dependency of random variables

Kullback Leibler divergence
Joint entropy
Negative marginal entropies

$$\begin{cases} X_1 \sim P_1 \\ X_2 \sim P_2 \end{cases} \qquad H_1(X_1)$$

Marginal PDF

Joint pdf:

$$P(X_1, X_2)$$

Product of marginal PDFs $q(X_1, X_2)$

$$= P_1(X_1)\, P_2(X_2)$$

Quasi-distance between joint PDF and product of marginal PDFs

$$D(P \| q)$$

$$D(P \| Q)$$

$$\neq D(Q \| P)$$

Quasi-distance

$$D(P \,||\, Q)$$

$$= \int p(x) \ln \frac{p(x)}{q(x)} dx$$

$$X = (x_1, X_2)$$

$$q(x) = P_1(X_1) P_2(X_2)$$

S

sample space

$\{1, 2, \ldots, 6\}$

$$\sum_{s} p(s) f(s) \triangleq E[f(s)]$$

$$p(x) = q(x)$$

$$D(p \| q)$$

$$= \int p(x) \ln \frac{p(x)}{q(x)} dx$$

$$= \int p(x) \ln 1 \, dx$$

$$= 0$$

Joint entropy

$$H(x) = -\int p(x) \ln p(x)\, dx$$

Summation of negative marginal entropies

$$= \sum_i H_i(x_i)$$

$$H_i(x_i) = -\int p_i(x_i) \ln p_i(x_i)\, dx_i$$

$$q = \prod_i P_i$$

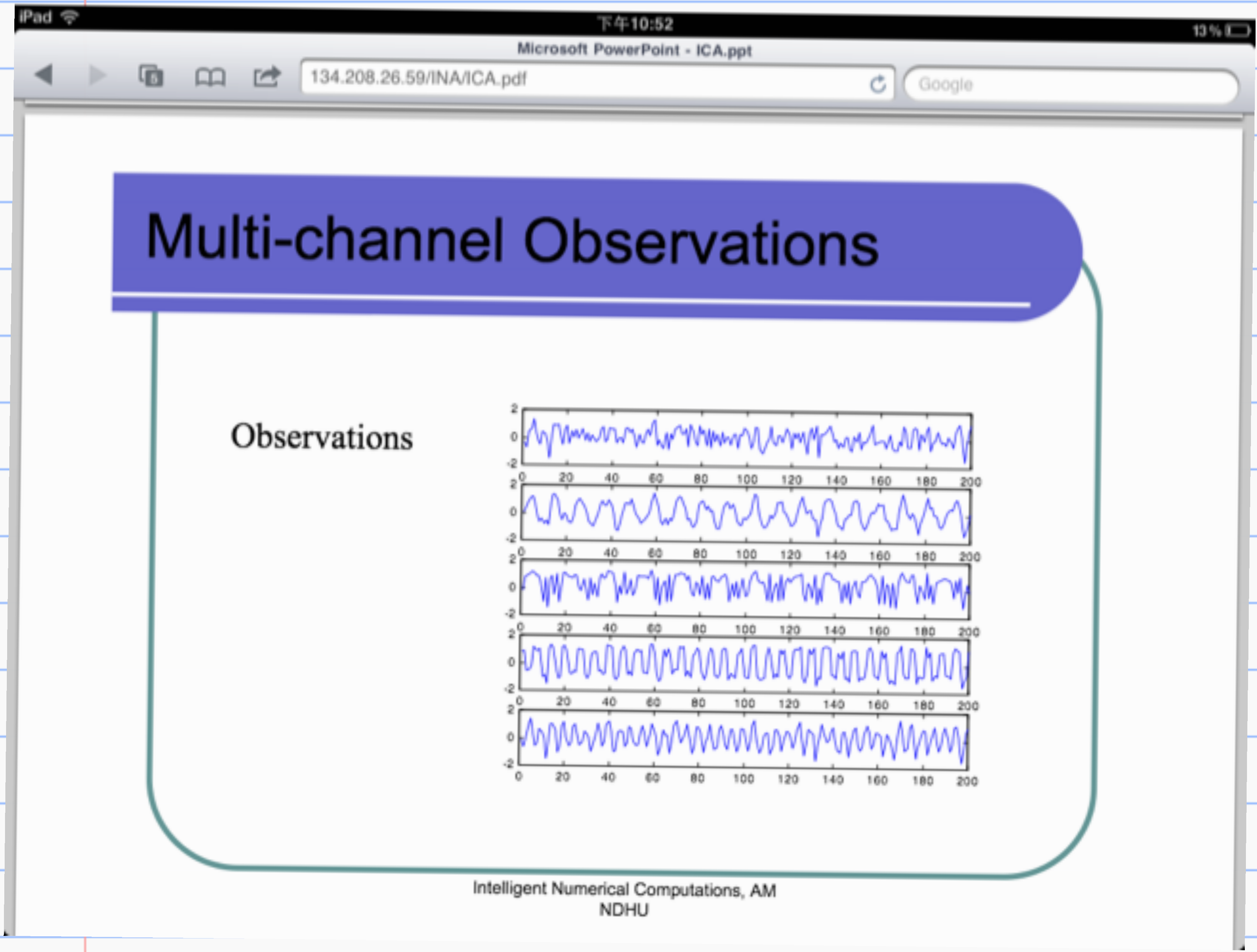
$$D(P \| q) = -\boxed{H(X)} + \boxed{\sum_i H_i(X_i)}$$

Maximal joint entropy
Minimal marginal entropies

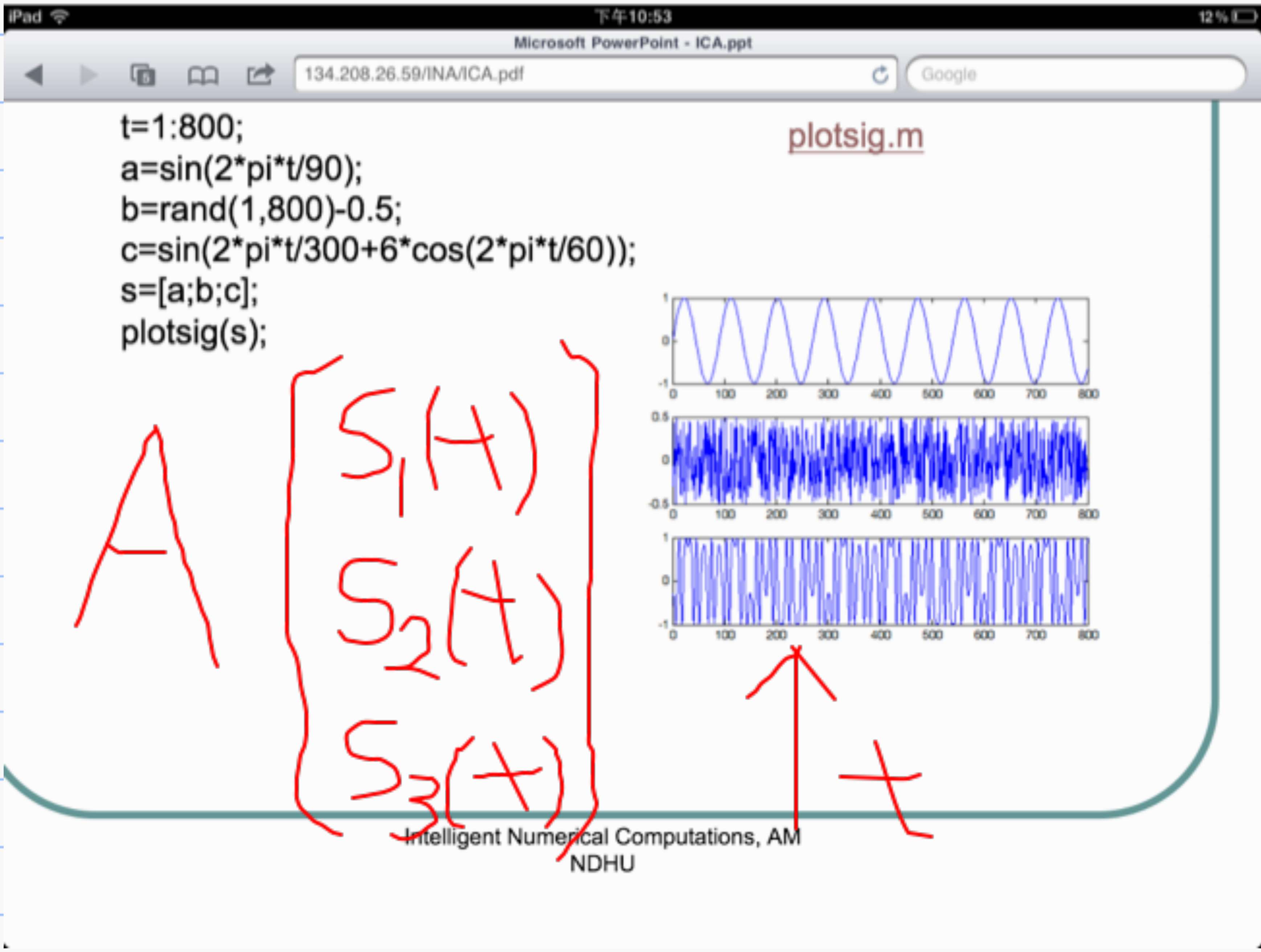
Marginal density estimation

- parametric density estimation
- semi-parametric density estimation
- histogram
- Parzen windows

Multichannel observations



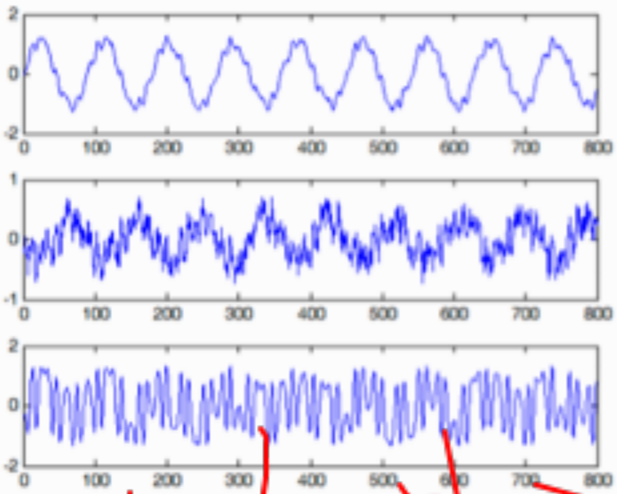
Three independent sources



Linear mixtures of independent sources

```
A=eye(3)*0.8+(rand(3,3)-0.5)*0.7;
x=A*s;
plotsig(x)
```

$$\mathbf{X}[t] = \mathbf{As}[t]$$



Linear Convolutive Mixture

Problem statement

• linear mixture assumption

• given multichannel observations are linear mixtures of independent components

• demixing process

• an effective demixing matrix whose product to multichannel observations leads to independent components