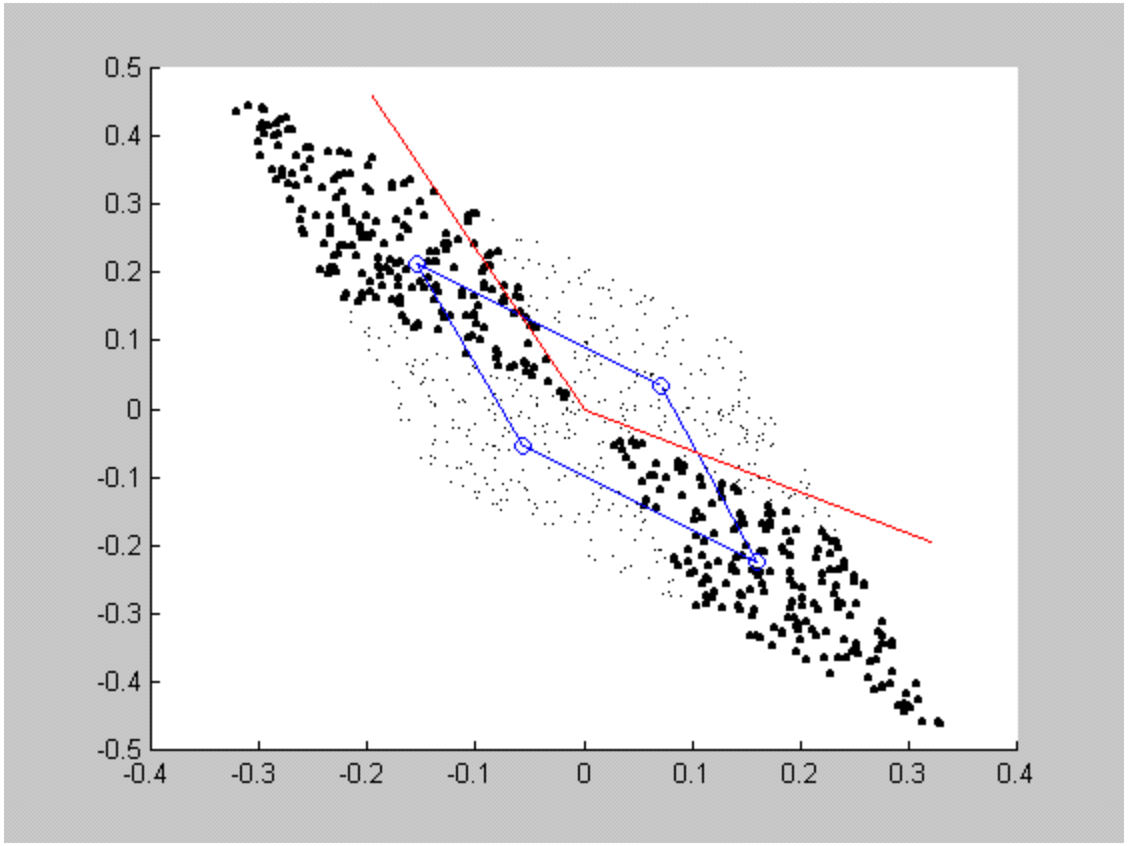
Independent component analysis

Bio-signal analysis

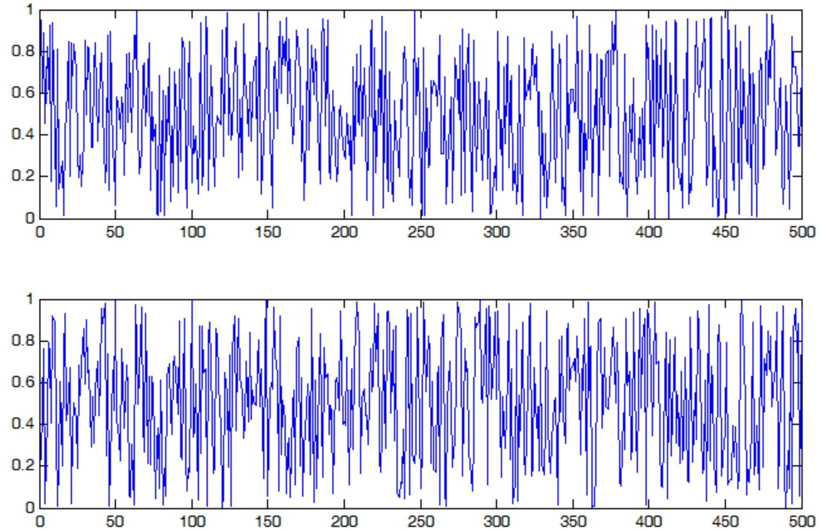
Statistical dependency



Two components

- Two independent sources

```
d=2;N=500;
s=rand(2,500);
subplot(2,1,1)
plot(1:1:N,s(1,:));
subplot(2,1,2)
plot(1:1:N,s(2,:));
```

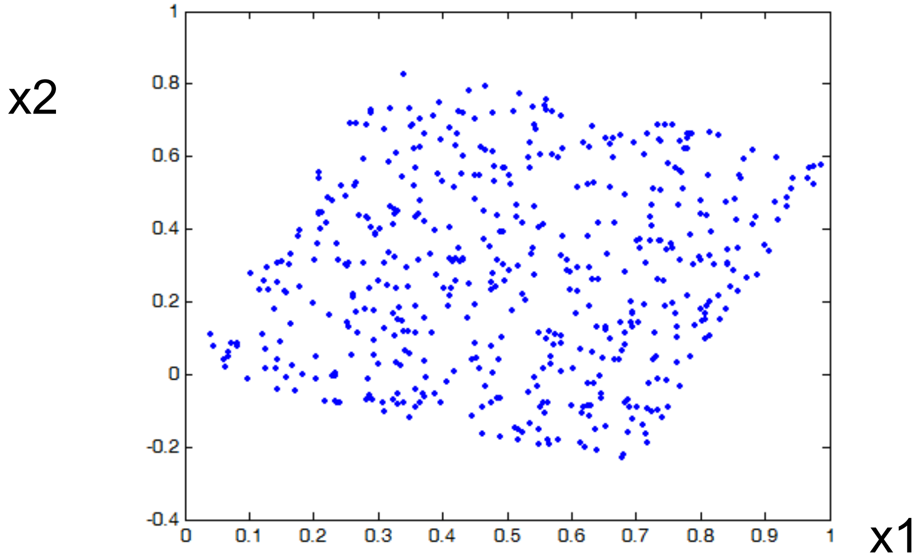


Linear mixtures

- Instant mixing

$$\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \mathbf{A} \begin{bmatrix} s_1(t) \\ s_2(t) \end{bmatrix}$$

```
A=[0.7 0.31;-0.25 0.85];
x=A*s;
plot(x(1,:),x(2,:),'.');
```



Stochastic modeling of sources

- S_1 and S_2 are independent random variables
- S_1 and S_2 represent independent sources iff

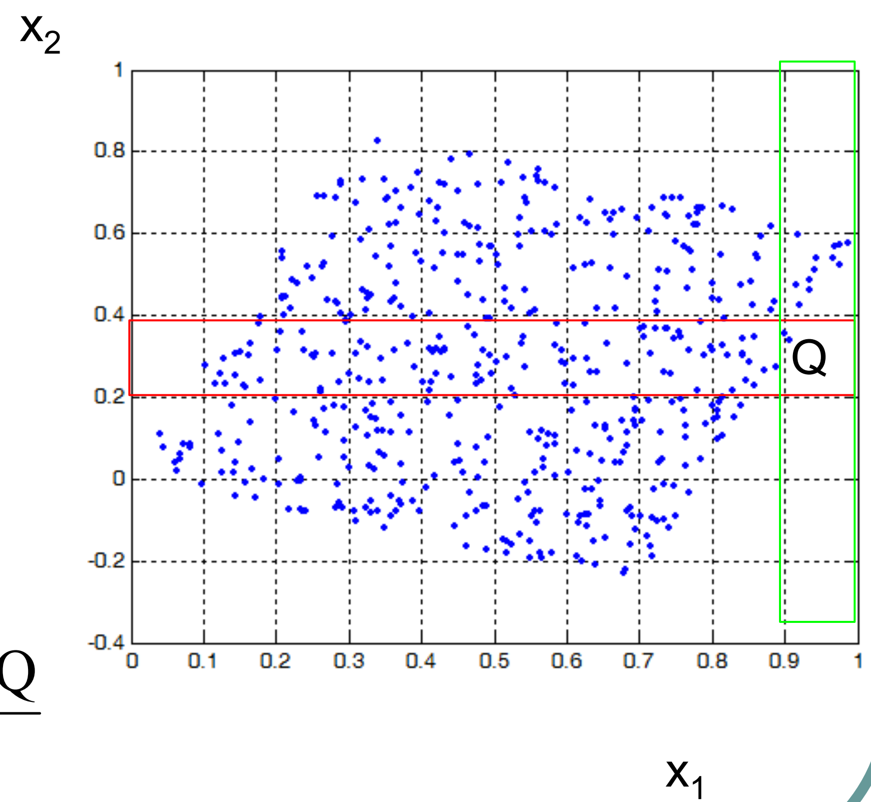
$$p(s_1, s_2) = p_1(s_1) p_2(s_2)$$

  - p: joint pdf
  - p_1, p_2: marginal pdfs

Rotated uniform sample

- Statistical dependent components
  - N: all data number
  - N_1: data number within red rectangle
  - N_2: data number within green rectangle

$$\frac{N_1 N_2}{N^2} \neq \frac{\text{number of data within square Q}}{N}$$



Artificially created independent sources

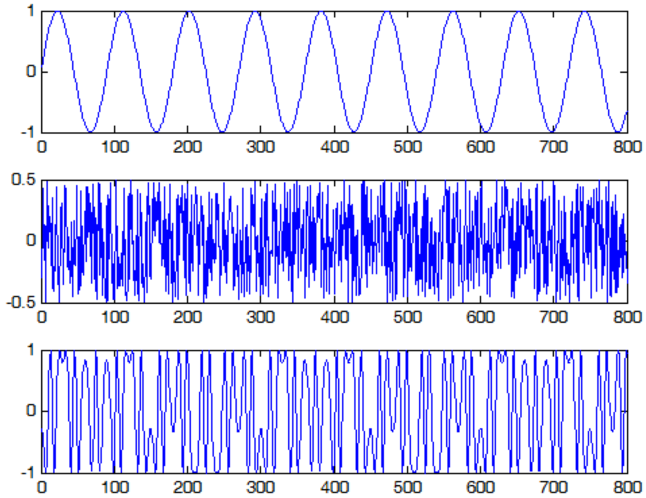
$$\mathbf{s}[t] = \begin{bmatrix} sign(\cos(2\pi t/155)) \\ \sin(2\pi t/800) \\ \sin(2\pi t/300 + 6\cos(2\pi t/60)) \\ \sin(2\pi t/90) \\ r(t) \end{bmatrix}$$

r(t): uniform sample

Three-channel observations

```
t=1:800;
a=sin(2*pi*t/90);
b=rand(1,800)-0.5;
c=sin(2*pi*t/300+6*cos(2*pi*t/60));
s=[a;b;c];
plotsig(s);
```

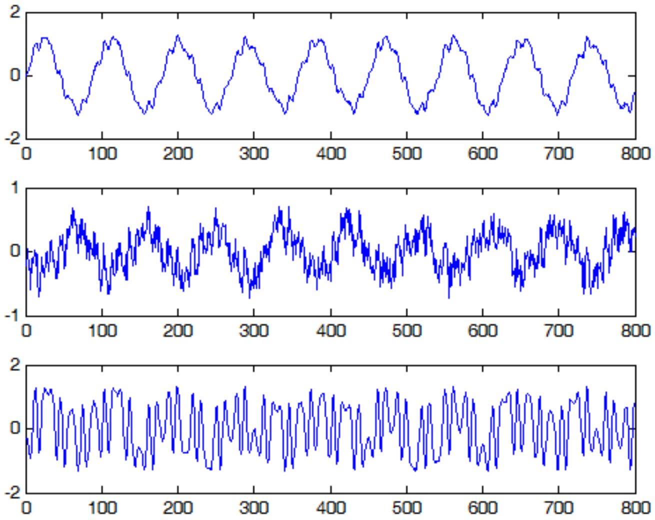
plotsig.m



Linear mixtures of independent sources

```
A=eye(3)*0.8+(rand(3,3)-0.5)*0.7;
x=A*s;
plotsig(x)
```

$$\mathbf{X}[t]=\mathbf{As}[t]$$

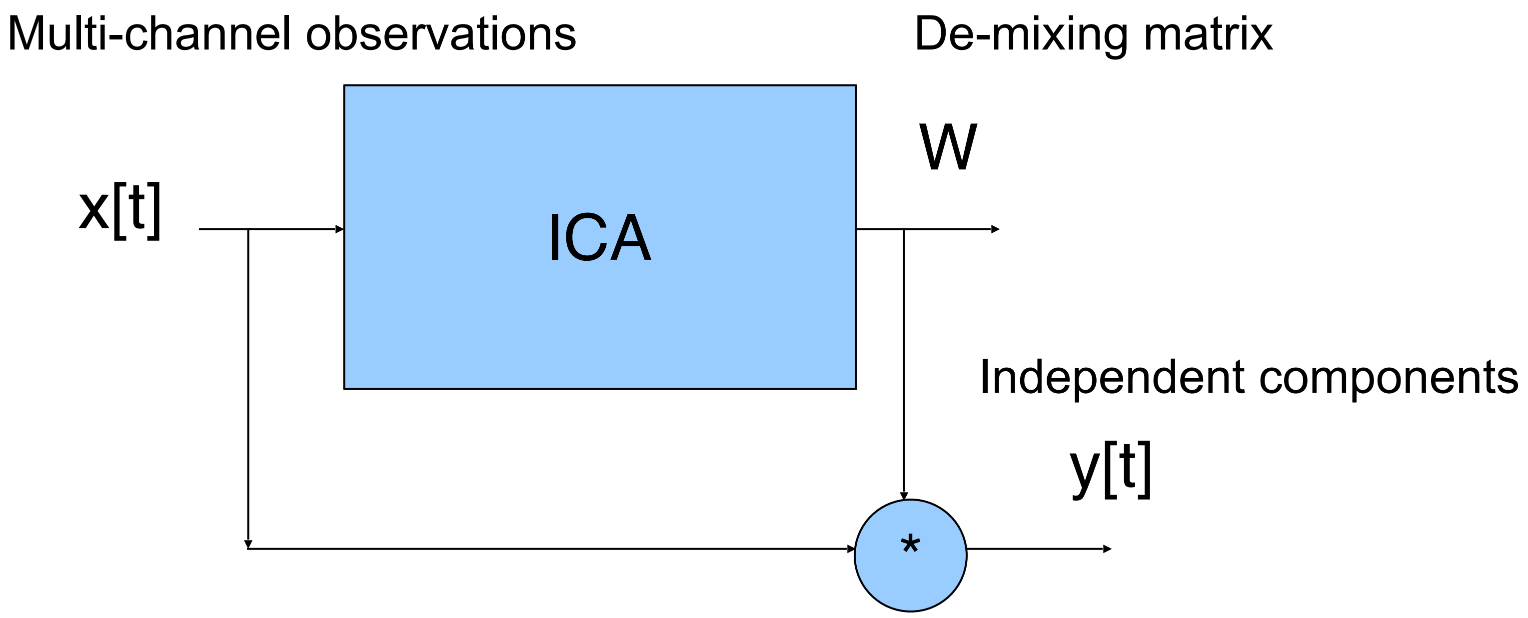


Independent component analysis

- Linear mixture assumption:
  Multi-channel observations are linear mixtures of independent sources

- Independent component analysis is aimed to recover independent sources for given multi-channel observations.

ICA algorithms



De-mixing

- An ICA algorithm returns a de-mixing matrix
- Multiplying the de-mixing matrix to multi-channel observations is expected to attain independent components

$$y[t] = Wx[t]$$

Fast Independent Component Analysis

FastICA

Independent Component Analysis: The Book

Publications by Aapo Hyvarinen: FastICA

Negative entropy

$$J(\mathbf{y}) = H(\mathbf{y}_{gauss}) - H(\mathbf{y})$$

Differential entropy

$$H(\mathbf{y}) = -\int f(\mathbf{y}) \log f(\mathbf{y}) d\mathbf{y}$$

$\mathbf{y}_{gauss}$ is a Gaussian vector of the same covariance matrix as $\mathbf{y}$

An approximation to negative entropy

Cumulant - based approximation

$$J(y) \approx c\left[E(G(y)) - E(G(v))\right]^2$$

y : zero mean and unit variance

v : a Gaussian variable of zero mean and unit variance

FastICA

Find one IC by maximizing

$$\mathrm{J}(\mathbf{w}) = [E\{\mathrm{G}(\mathbf{w}^T\mathbf{x})\} - E\{\mathrm{G}(v)\}]^2$$

constrained by

$$\mathrm{E}\{(\mathbf{w}^T\mathbf{x})^2\} = 1$$

$\mathbf{w}$: an $m \times 1$ vector

Nonlinear equations

- Let

$$G(y) = \frac{1}{4} y^4,$$

$$g(y) = \frac{dG}{dy} = y^3$$

- Maximizing J_G(w) is realized by solving

$$E\{\mathbf{x} g(\mathbf{w}^T \mathbf{x})\} - \beta \mathbf{w} = 0$$

$$\beta = E\{\mathbf{w}_0^T \mathbf{x} g(\mathbf{w}_0^T \mathbf{x})\}$$

$\mathbf{w}_0$ is the value of $\mathbf{w}$ at the optimum

Jacobi matrix

$$F(\mathbf{w}) = E\{\mathbf{x}g(\mathbf{w}^T\mathbf{x})\} - \beta\mathbf{w}$$

$$J(\mathbf{w}) = \frac{dF}{d\mathbf{w}} = E\{\mathbf{x}\mathbf{x}^T g'(\mathbf{w}^T\mathbf{x})\} - \beta I$$

$$= E\{\mathbf{x}\mathbf{x}^T\}E\{g'(\mathbf{w}^T\mathbf{x})\} - \beta I$$

$$= E\{g'(\mathbf{w}^T\mathbf{x})\}I - \beta I \qquad \text{if } E\{\mathbf{x}\mathbf{x}^T\} = I$$

Newton's method

- Updating rule

$$\mathbf{w}^{+} = \mathbf{w} - \mathrm{F}(\mathbf{w})J^{-1}(\mathbf{w})$$

$$= \mathbf{w} - \left[\mathrm{E}\{\mathbf{x}g(\mathbf{w}^{\mathrm{T}}\mathbf{x})\} - \beta\mathbf{w}\right] / \left[\mathrm{E}\{g'(\mathbf{w}^{\mathrm{T}}\mathbf{x})\} - \beta\right]$$

$$normalization:$$

$$\mathbf{w}^{*} = \mathbf{w}^{+} / \left\|\mathbf{w}^{+}\right\|$$

$$\beta = \mathrm{E}\{\mathbf{w}^{T}\mathbf{x}g(\mathbf{w}^{T}\mathbf{x})\}$$

Find multiple ICs

- We have $w_1, w_2, \ldots, w_{p-1}, w_p$
- We want to find the p+1 vector
  - Use updating rule to find $\mathbf{w}^*$ and set $\mathbf{w}_{p+1} = \mathbf{w}^*$
  - Set

$$\mathbf{w} = \mathbf{w} - \sum_{k=1}^{p} \mathbf{w}_k^T \mathbf{w} \mathbf{w}_k$$

$$\mathbf{w}_{p+1} = \mathbf{w} / \|\mathbf{w}\|$$

  - Go to step 1

FastICA Package

- Download FastICA
- Unpack FastICA
- Add FastICA_2.5 to the path
- Run fasticag
  - load x
  - Plot data
  - Do ICA

ICA recent advances

- Independent component analysis : recent advances

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Blind Separation of Time/Position Varying Mixtures

Ran Kaftory and Yehoshua Y. Zeevi

***Abstract*—We address the challenging open problem of blindly separating time/position varying mixtures, and attempt to separate the sources from such mixtures without having prior information about the sources or the mixing system. Unlike studies concerning instantaneous or convolutive mixtures, we assume that the mixing system (medium) is varying in time/position. Attempts to solve this problem have mostly utilized, so far, online algorithms based on tracking the mixing system by methods previously developed for the instantaneous or convolutive mixtures. In contrast with these attempts, we develop a unified approach in the form of staged sparse component analysis (SSCA). Accordingly, we assume that the sources are either sparse or can be "sparsified." In the first stage, we estimate the filters of the mixing system, based on the scatter plot of the sparse mixtures' data, using a proper clustering and curve/surface fitting. In the second stage, the mixing system is inverted, yielding the estimated sources. We use the SSCA approach for solving three types of mixtures: time/position varying instantaneous mixtures, single-path mixtures, and multipath mixtures. Real-life scenarios and simulated mixtures are used to demonstrate the performance of our approach.**

***Index Terms*—Blind source separation (BSS), sparse component analysis (SCA), time/osition varying mixing/unmixing.**

be statistically independent. This approach lends itself to a geometric interpretation of the mixing coefficients, whereby the mixing matrix entries can be retrieved from the scatter plot of the sparsified mixtures [9].

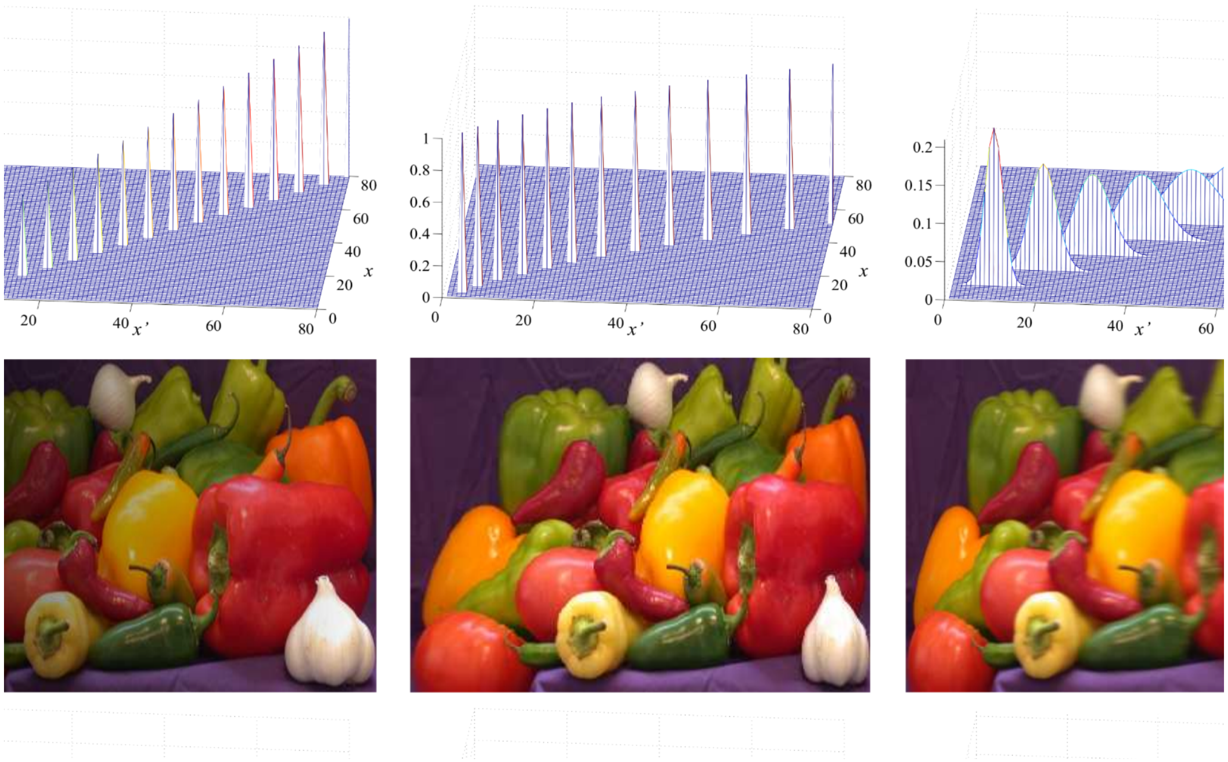
In most real-life scenarios, the mixing system is not constant as is in the case of instantaneous or convolutive model. It is varying as a function of time or position. For example, the attenuation of signals/images varies over time/position thus creating time/position varying instantaneous mixtures. The delay/shift or reverberation/blurring of a signal/image may also vary over time/position, creating a time/position varying single/multipath mixtures. Only few studies address this generalized BSS problem. Most of them use the ICA approach and assume a slow varying mixing system, thus, enabling the use of an adaptive version of the algorithms developed for the stationary cases.

In this paper, we extend and generalize the BSS problem and provide a unified approach to blind separation of certain classes of time/position varying mixtures that have not been dealt with so far. To this end, we present a framework of staged SCA (SSCA).

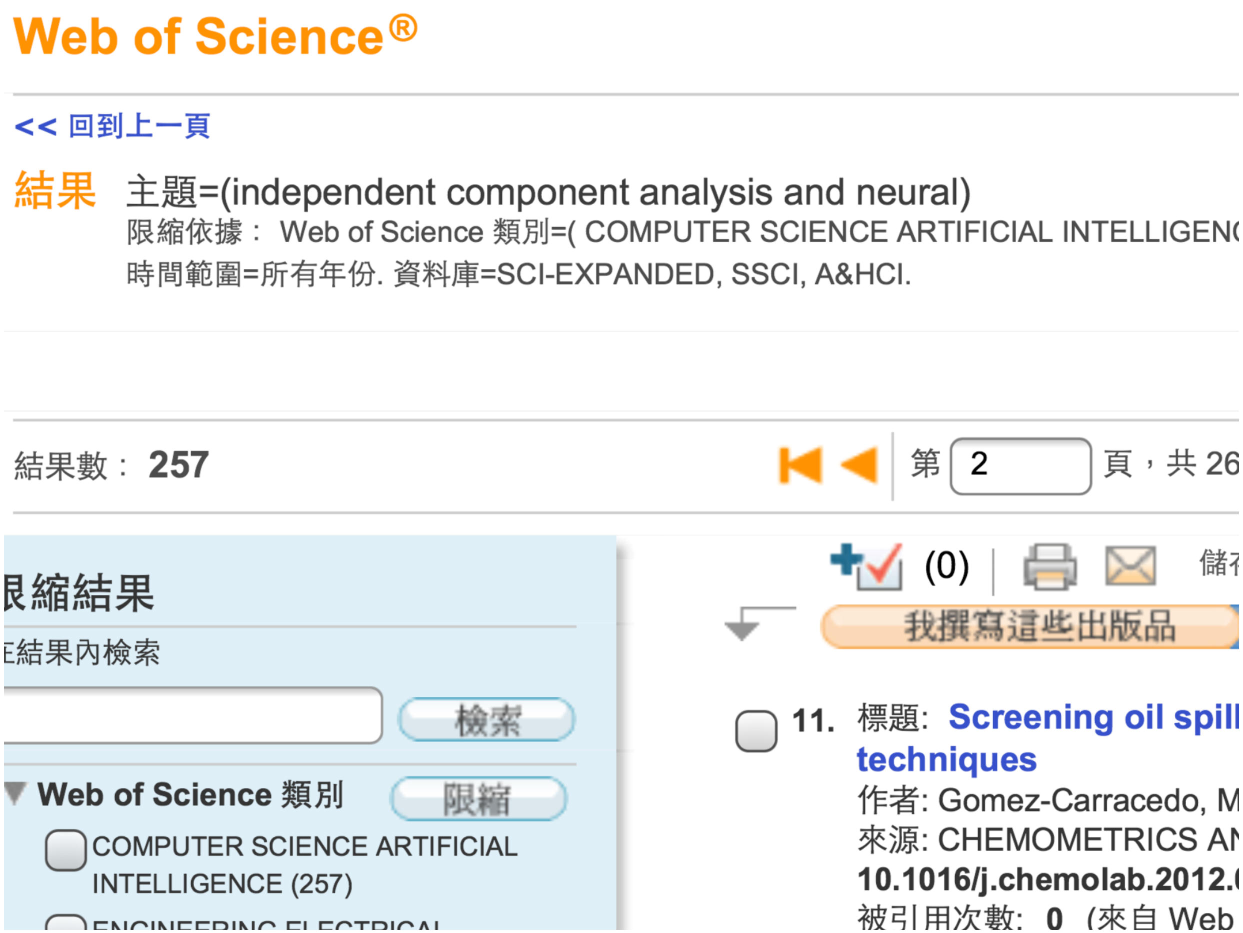
Blind separation of time/position varying mixtures

- pdf

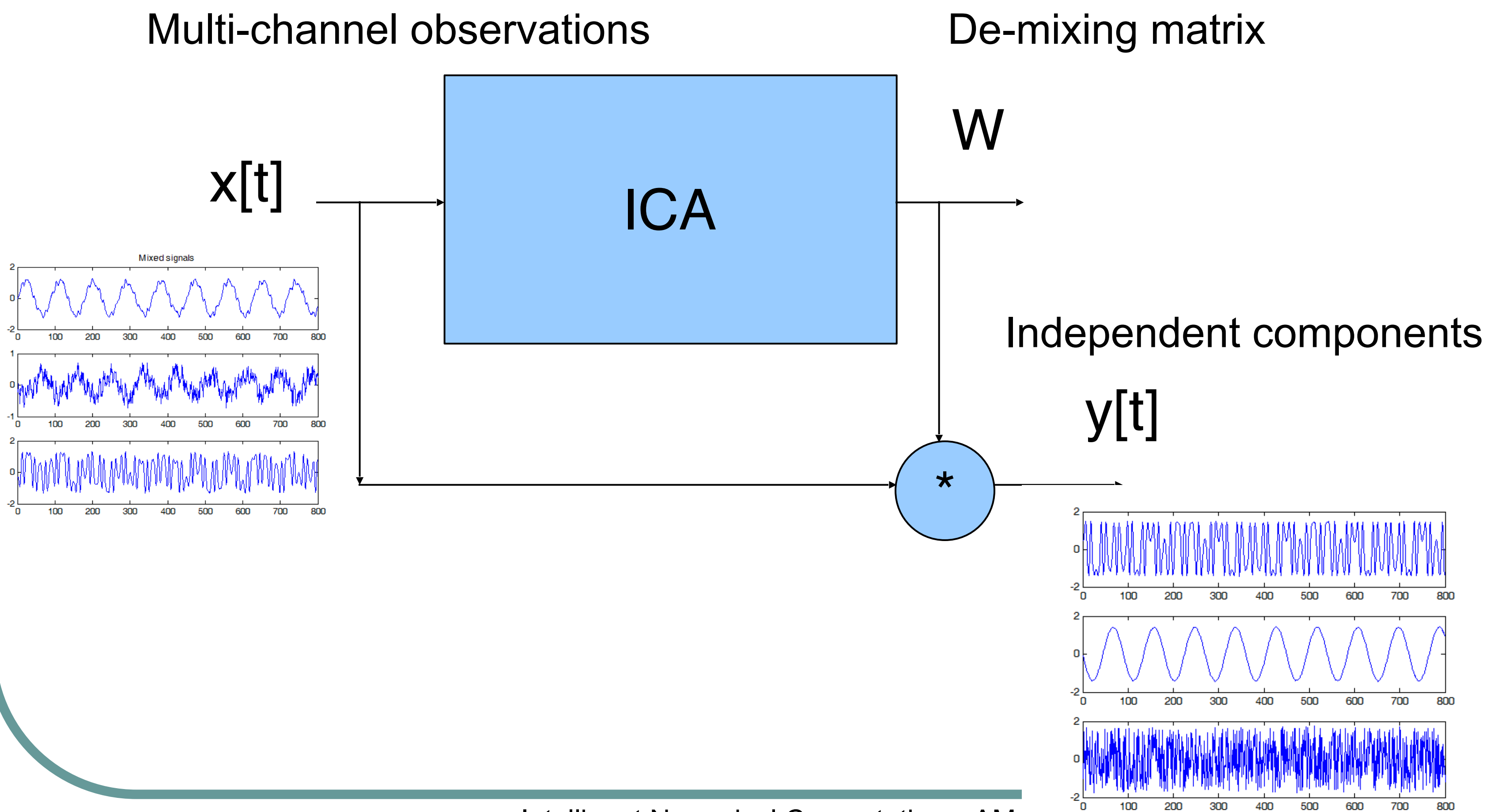
ND ZEEVI: BLIND SEPARATION OF TIME/POSITION VARYING MIXTURES



Independent component analysis

- 

FastICA


Multi-channel observations
De-mixing matrix
W
x[t]
ICA
Mixed signals
Independent components
y[t]
*

Five sources

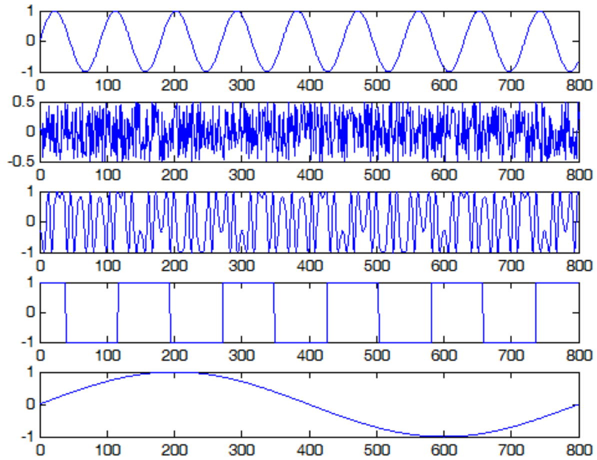
$$\mathbf{s}[t] = \begin{bmatrix} sign(\cos(2\pi t/155)) \\ \sin(2\pi t/800) \\ \sin(2\pi t/300 + 6\cos(2\pi t/60)) \\ \sin(2\pi t/90) \\ r(t) \end{bmatrix}$$

r(t): uniform sample

Five-channel observations

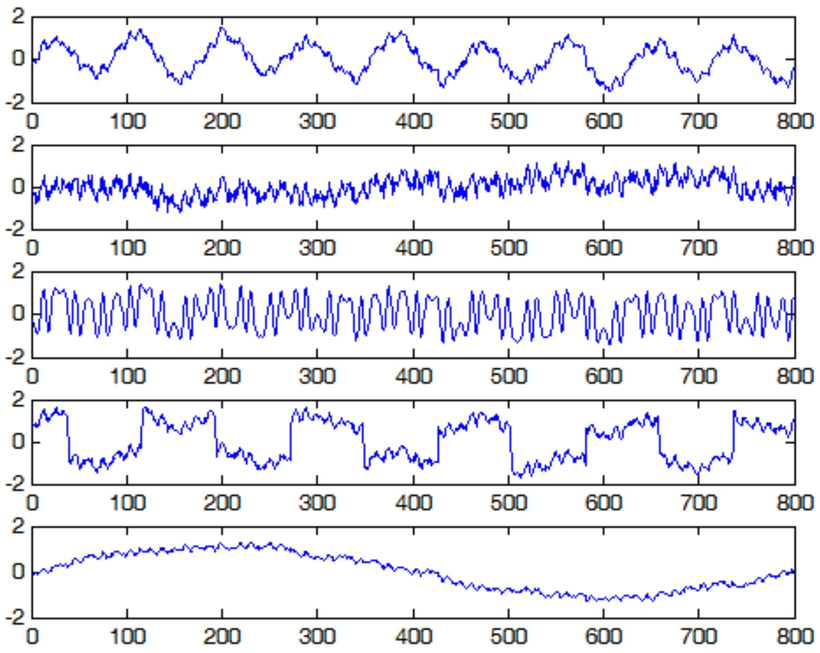
```
t=1:800;
a=sin(2*pi*t/90);
b=rand(1,800)-0.5;
c=sin(2*pi*t/300+6*cos(2*pi*t/60));
d=sign(cos(2*pi*t/155));
e=sin(2*pi*t/800);
s=[a;b;c;d;e];
plotsig(s);
```

plotsig.m



Linear mixtures of independent sources

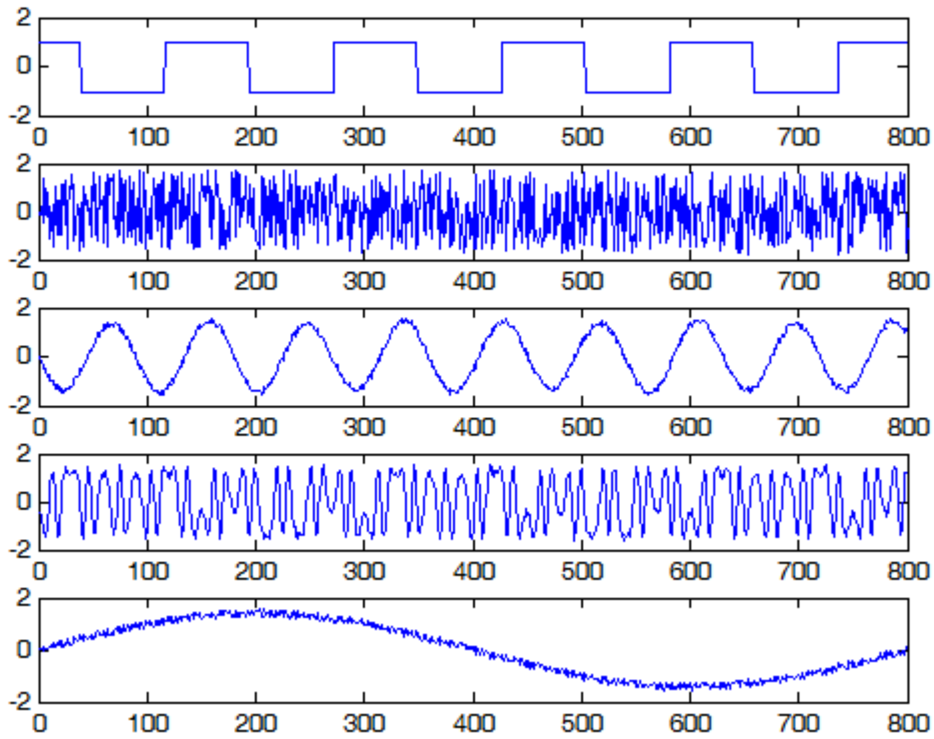
```
A=eye(5)*0.8+(rand(5,5)-0.5)*0.7;
x=A*s;
plotsig(x)
```



$$\mathbf{X}[t]=\mathbf{A}\mathbf{s}[t]$$

Recovered Independent components

ICs derived by FastICA



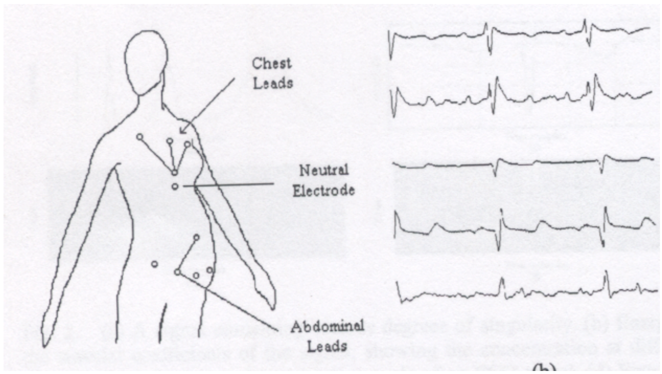
ICA algorithms

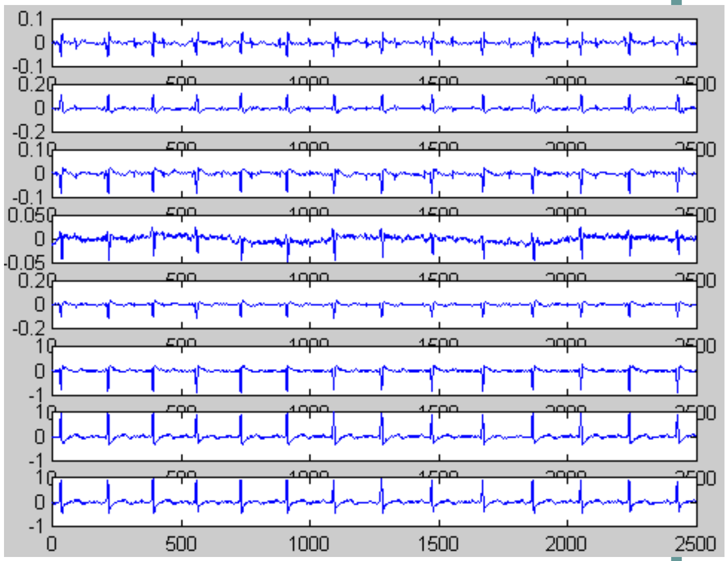
JadeICA

FastICA

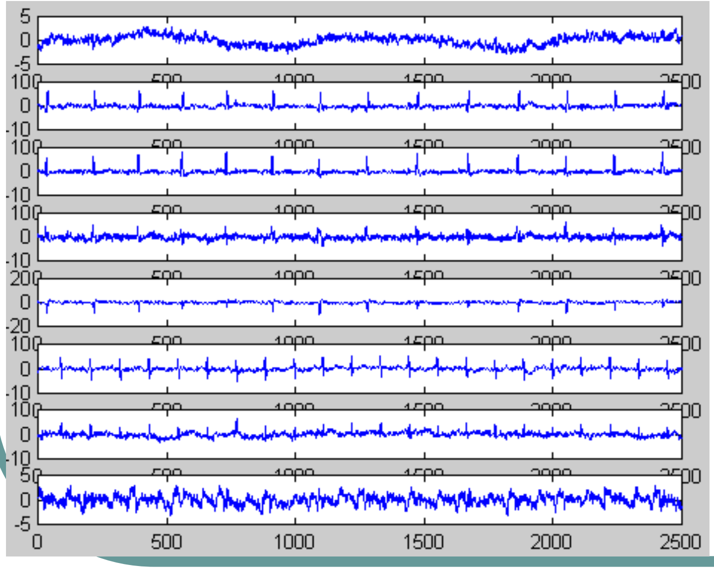
PottsICA: Kullback-Leibler Divergence

Blind source separation – fetal ECG





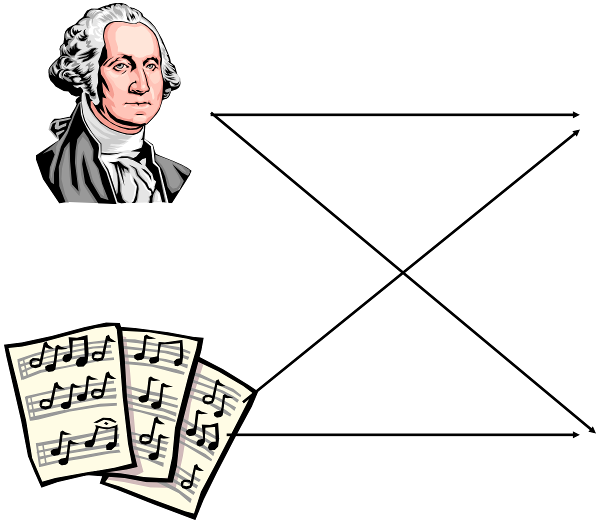
BSS

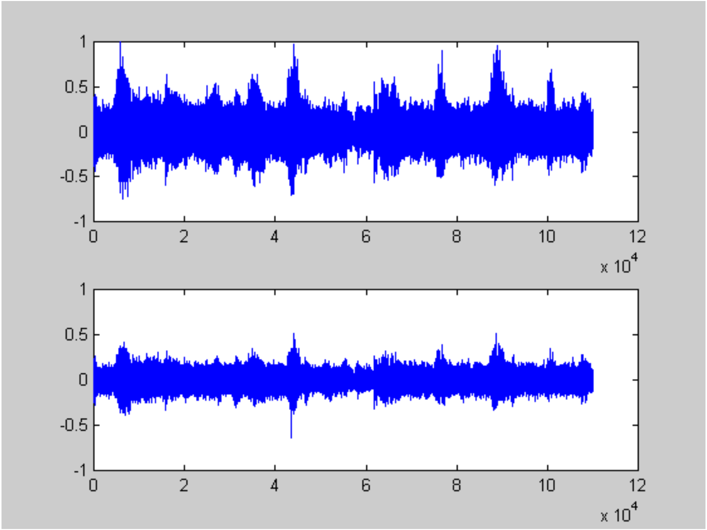



sources
mixed images
AemICA
JadeICA

Sound separation

music and speech

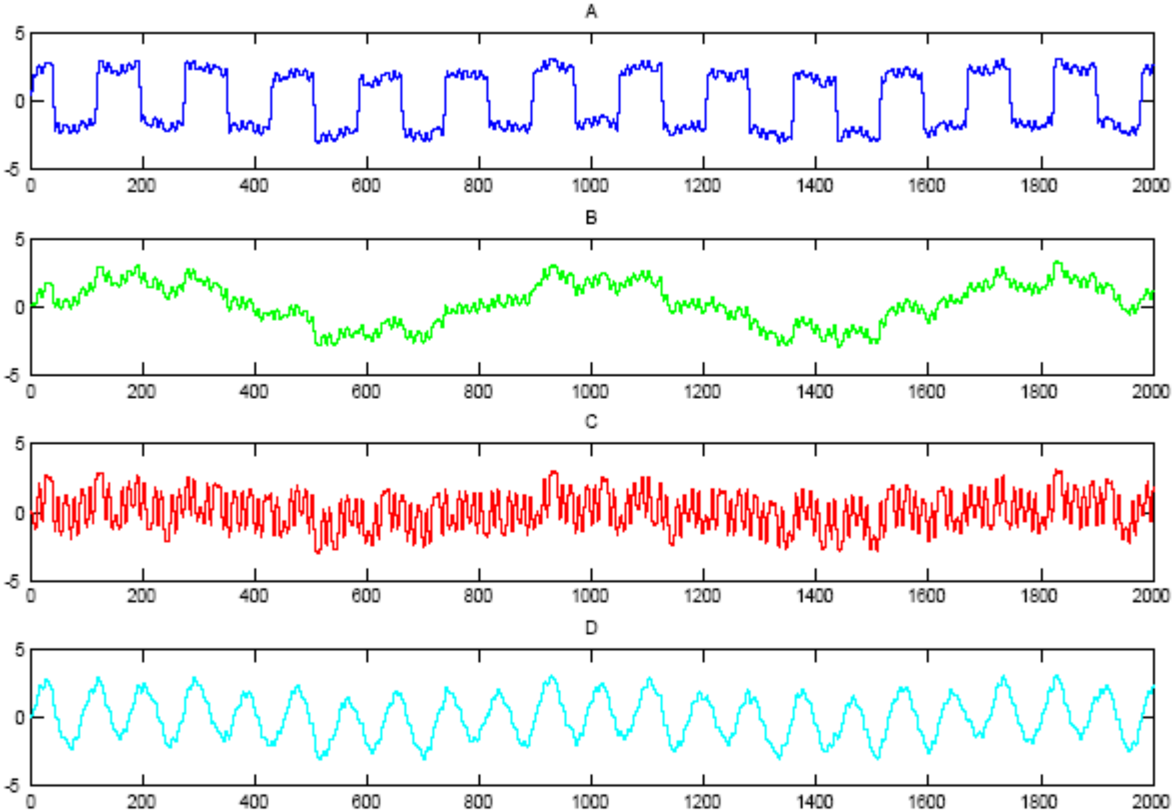




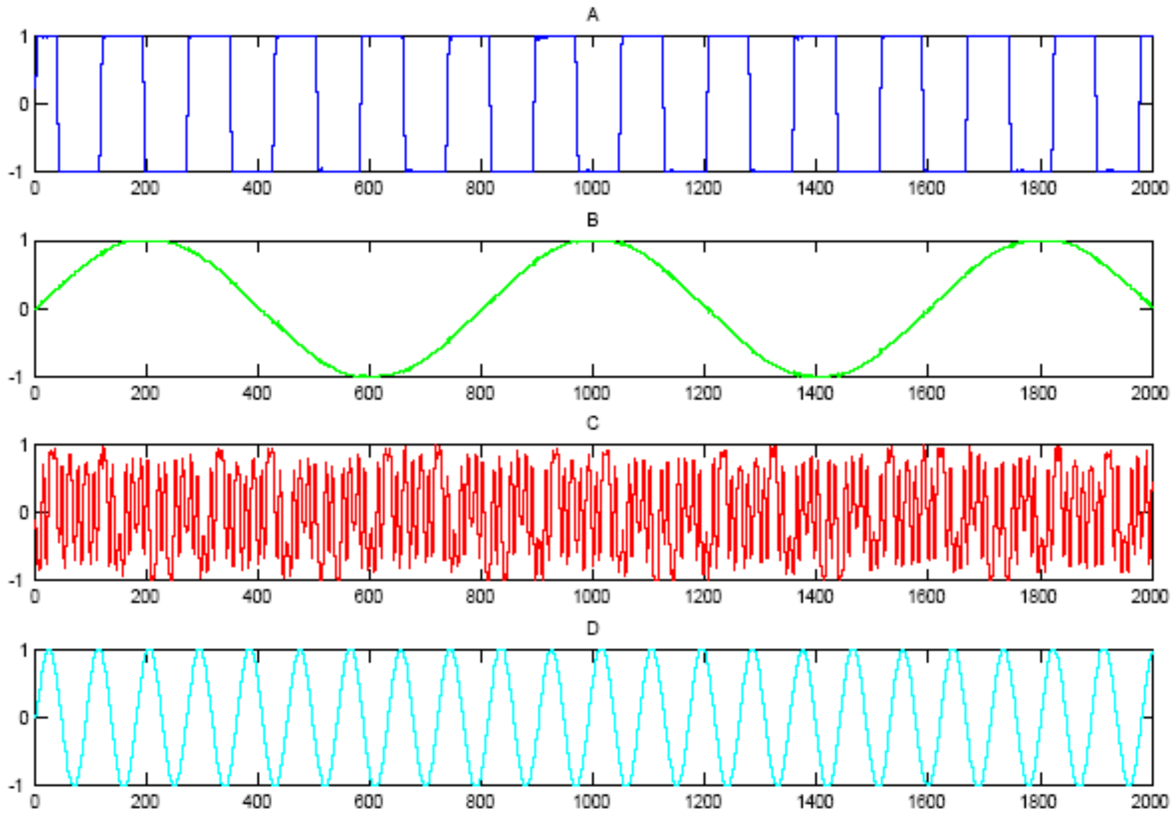

(.wav)

Convolutive mixtures

$\tau = 5$



Recovered sources



Blind separation of real world signals

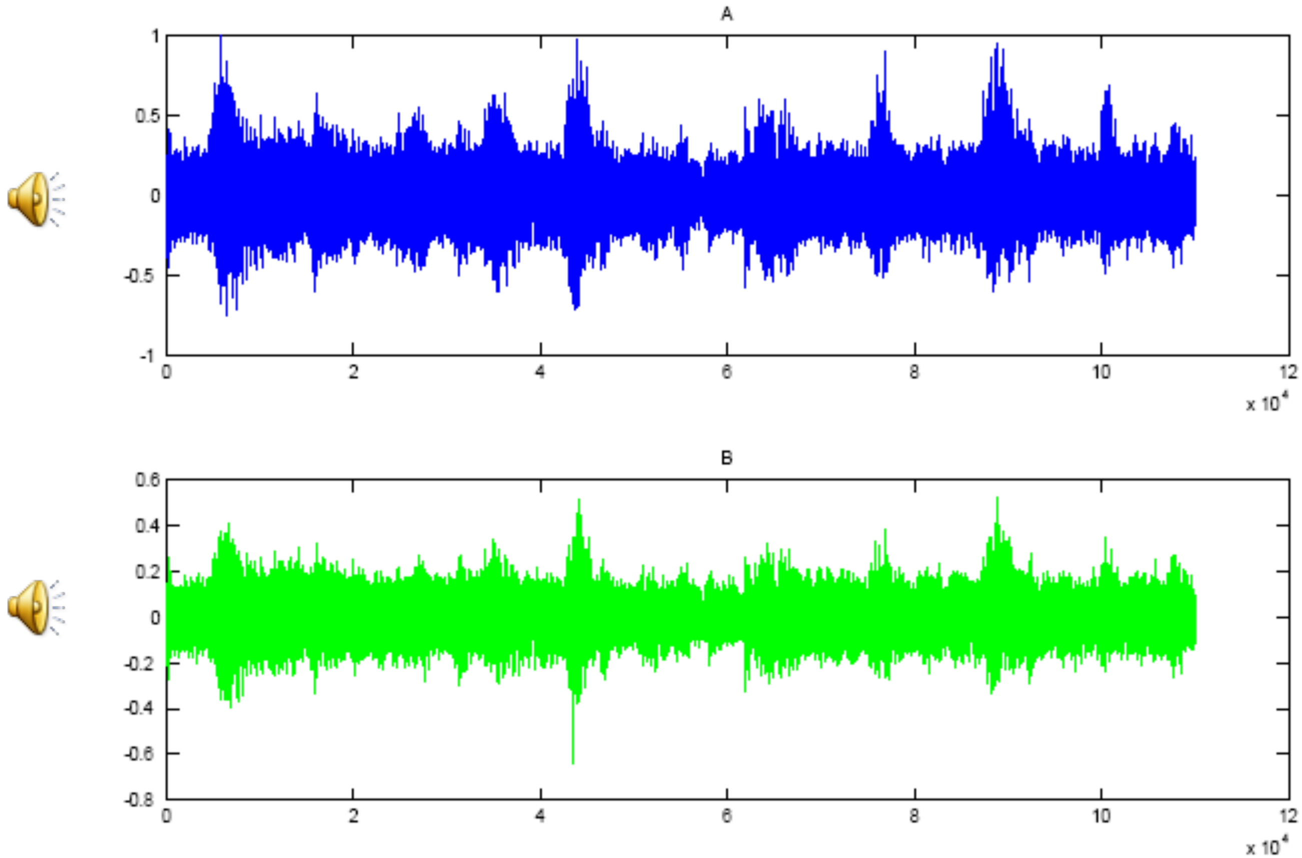
Blind separation of real world signals

The experiment results as follows:

- Two-microphone recordings of music and speech.
  - Channel-1 Sound
  - Channel-2 Sound
- Blind separation of recordings of music and speech.
  - Channel-1 Sound
  - Channel-2 Sound

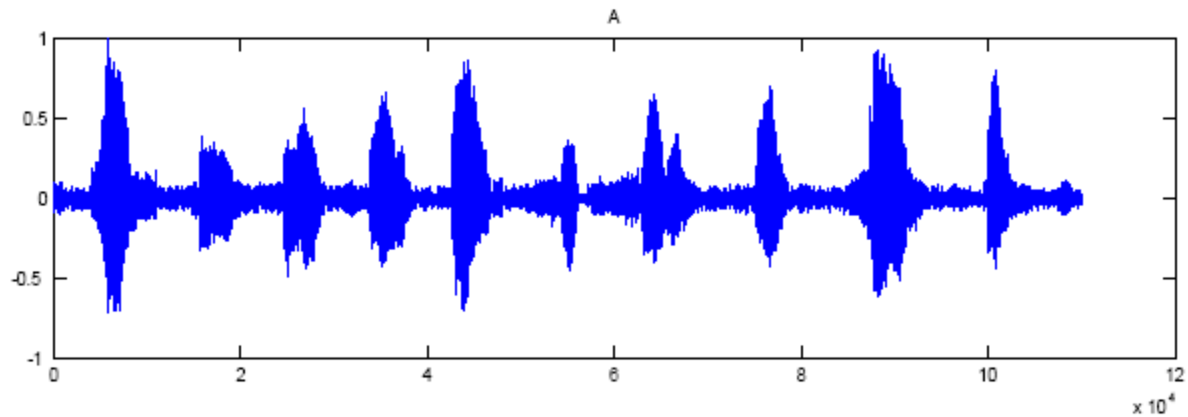
Demo

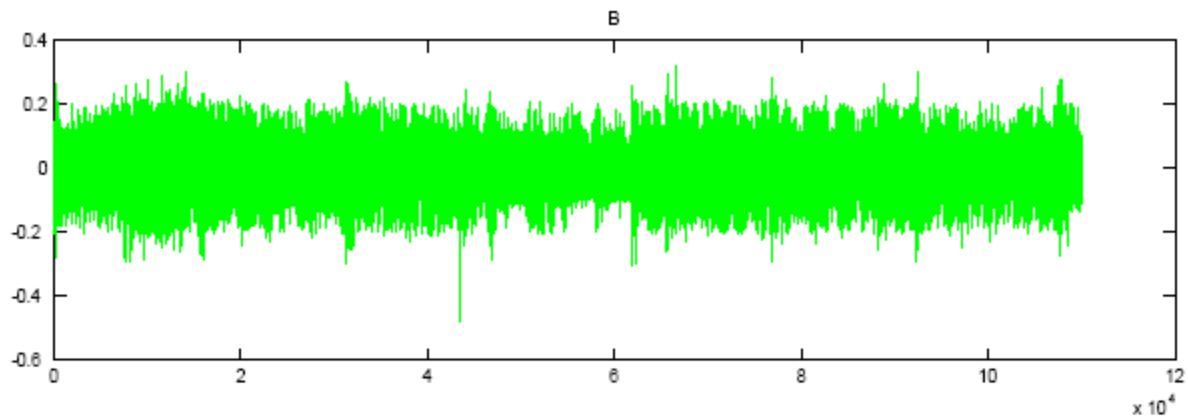
The recordings of two microphones are shown in the following figure.



Demo

The blind separation of music and speech are shown in the following figure.





JadeICA

Exercise:

- Download JadeICA package
- Apply JadeICA to deal with three-channel observations
- Apply JadeICA to deal with five-channel observations

Facial images

- Form linear mixtures of five facial images and a noise image
- Apply JadeICA to recover facial images


sources
mixed images
AemICA
JadeICA