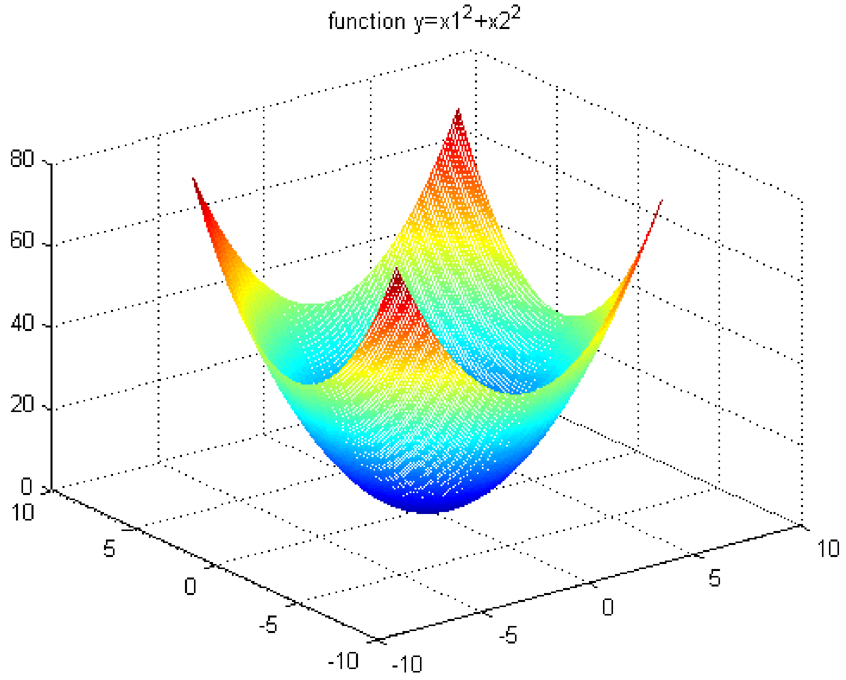
A 2-dimension function Example 1

$$f(x_1, x_2) = x_1^2 + x_2^2$$

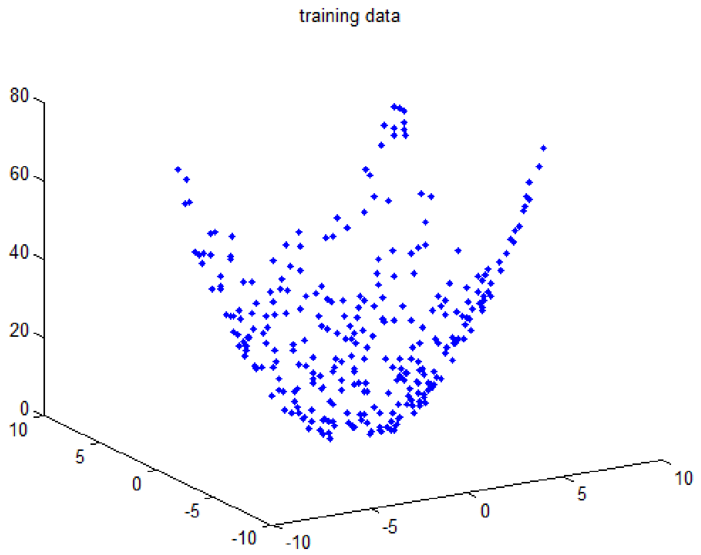
fa_2d.m

```
fstr=input('input a 2D function: x1.^2+x2.^2+cos(x1) :','s');
fx=inline(fstr);
range=2*pi;
x1=-range:0.1:range;
x2=x1;
for i=1:length(x1)
        C(:,i)=fx(x1(i),x2);
end
mesh(x1,x2,C);
```



Sampling training data

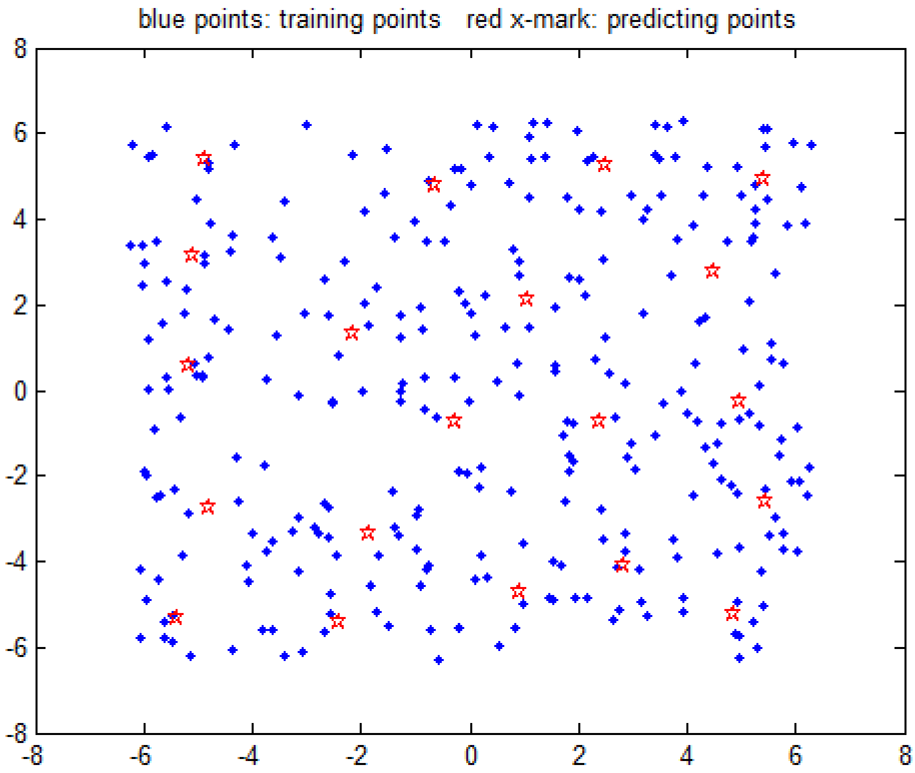
```
N=input('input the number of sample:');
x=rand(2,N)*4*pi-2*pi;
y=fx(x(1,:),x(2,:));
y=y+rand(1,N)*3-1.5;
N=length(x);
ind=randperm(N);
N2=floor(N/2);
X_TRAIN=x(:,ind(1:N2));
Y_TRAIN=y(ind(1:N2));
figure
plot3(X_TRAIN(1,:), X_TRAIN(2,:), Y_TRAIN,'.')
```



Step 1 Searching means

- Kmeans

```
K=20;
figure
plot(X_TRAIN(1,:),X_TRAIN(2,:),'b.')
[cind,center]=kmeans(X_TRAIN',K);
hold on
plot(center(:,1),center(:,2),'rP')
```



Step 2 Cross Distances

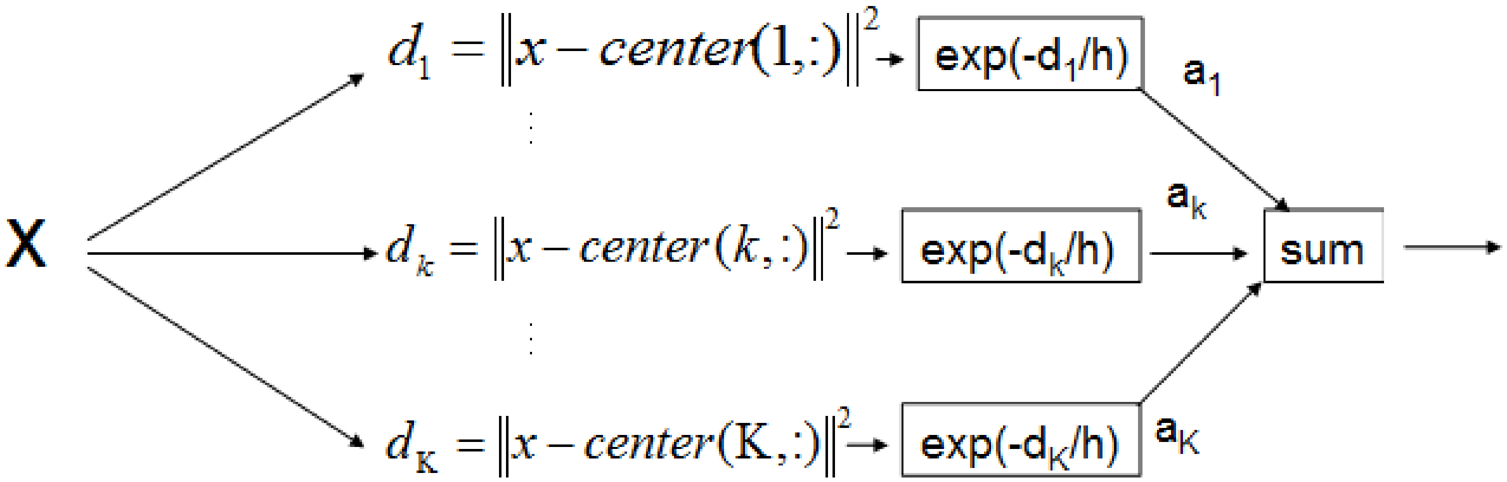
- Cross distances between centers and data points

```
D = cross_distance(X_TRAIN',center);
```

- D(i,j) stores the distance between the ith data point and the jth center

Step 3 Posterior weights

- Radial basis function



X

$$d_1 = \left\| x - center(1,:) \right\|^2$$

exp(-d_1/h)

a_1

$$d_k = \left\| x - center(k,:) \right\|^2$$

exp(-d_k/h)

a_k

sum

$$d_K = \left\| x - center(K,:) \right\|^2$$

exp(-d_K/h)

a_K

Approximating function

S*ubstitute* the ith data point

$$y(i) = f(x(i,:))$$

$$= \sum_{k=1}^{K} a_k \exp(-\left\| x(i,:) - center(k,:) \right\|^2 / h)$$

$$= \sum_{k=1}^{K} a_k \exp(-d_{ik} / h)$$

$$D = [d_{ik}], \mathbf{a} = \begin{bmatrix} a_1 \\ \vdots \\ a_k \\ \vdots \\ a_K \end{bmatrix}, \mathbf{y} = \begin{bmatrix} y_1 \\ \vdots \\ y_k \\ \vdots \\ y_K \end{bmatrix}$$

Substitute the ith data point

$$y(i) = f(x(i,:))$$

$$= \sum_{k=1}^{K} a_k \exp(-\|x(i,:) - center(k,:)\|^2)$$

$$= \sum_{k=1}^{K} a_k \exp(-d_{ik})$$

⇨ $\exp(-D/h)\mathbf{a} = \mathbf{y}$

$$\exp(-D/h)\mathbf{a} = \mathbf{y}$$

↓

$$\mathbf{a} = pinv(\exp(-D/h)) * \mathbf{y}$$

```
h=40;
a=pinv(exp(-D/h))*Y_TRAIN';
```

Approximating function

X_TRAN

centers

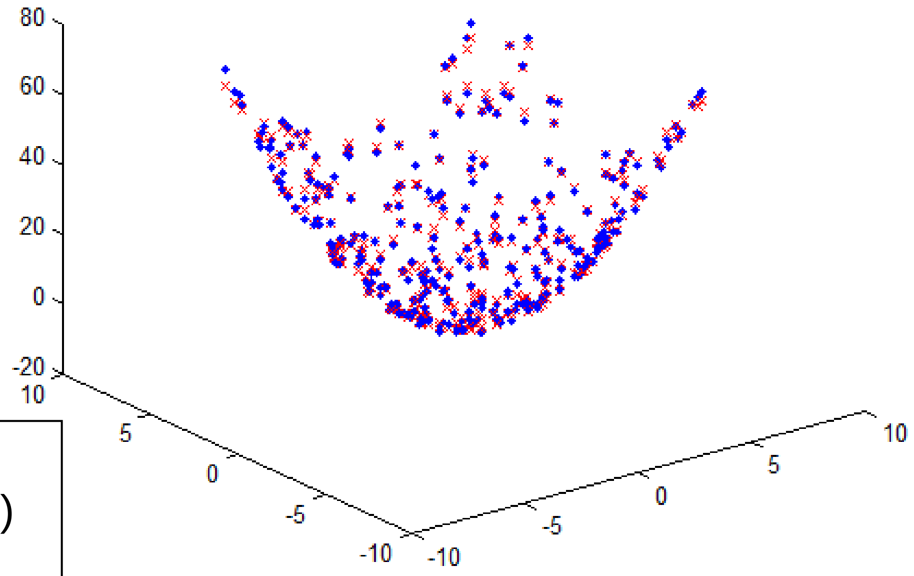
```
D = cross_distance(X_TRAIN',center);
```

a, h

```
Y_HAT=exp(-D/h)*a;
```

Y_HAT

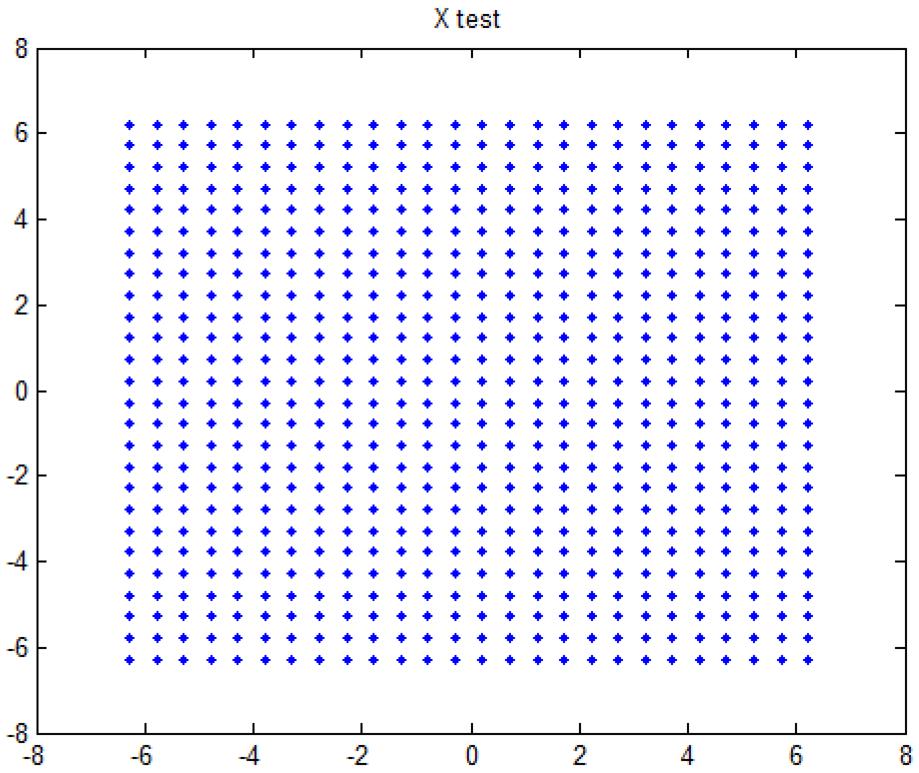
```
figure
plot3(X_TRAIN(1,:),X_TRAIN(2,:),Y_TRAIN,'.')
hold on
plot3(X_TRAIN(1,:),X_TRAIN(2,:),Y_HAT,'rx')
```



blue points: training points red x-mark: predicting points

Form X_test

```
range=2*pi;
x1=-range:0.5:range;
x2=x1;
X_test=[ ];
for i=1:length(x1)
    xx = [x1(i)*ones(1,length(x2));x2];
    X_test=[X_test xx];
end
figure
plot(X_test(1,:),X_test(2,:),'.')
```



X_test

centers

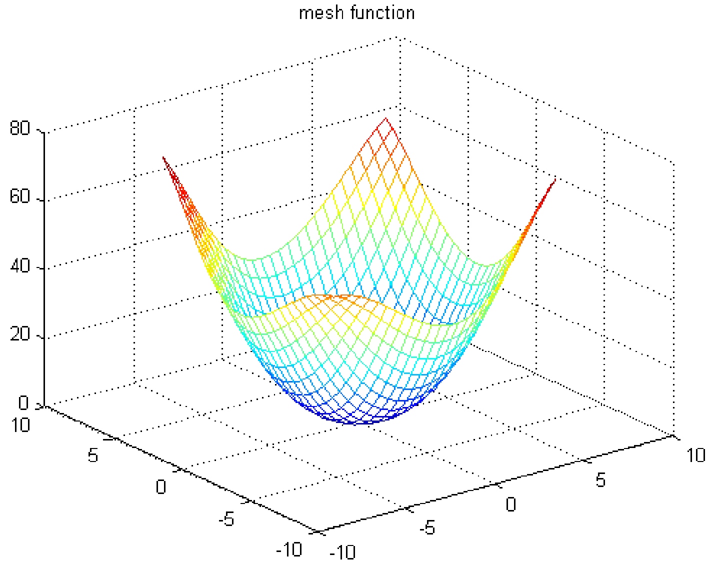
```
D = cross_distance(X_test',center);
```

a, h

```
Y_test=exp(-D/h)*a;
```

Y_test

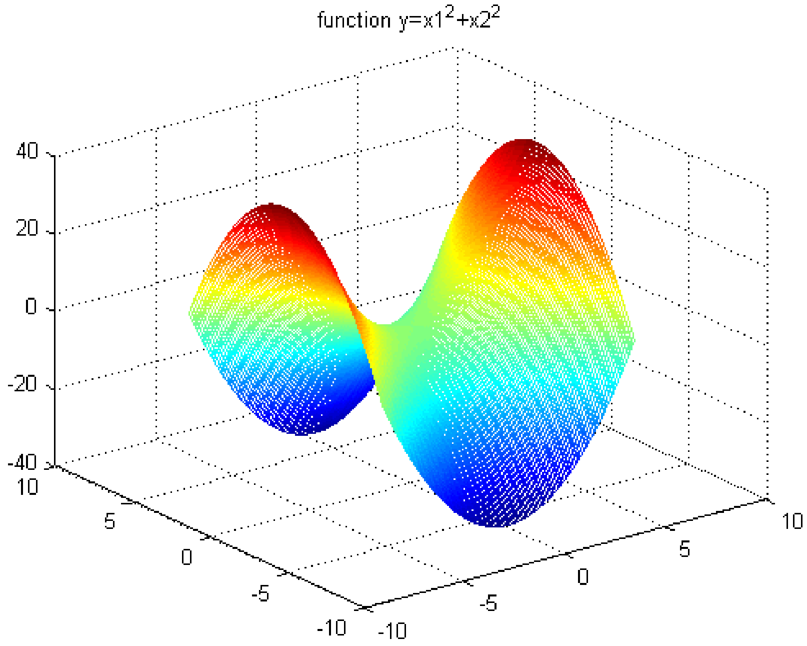
```
figure
CC=reshape(Y_test,length(x1),length(x1));
mesh(x1,x2,CC);
```



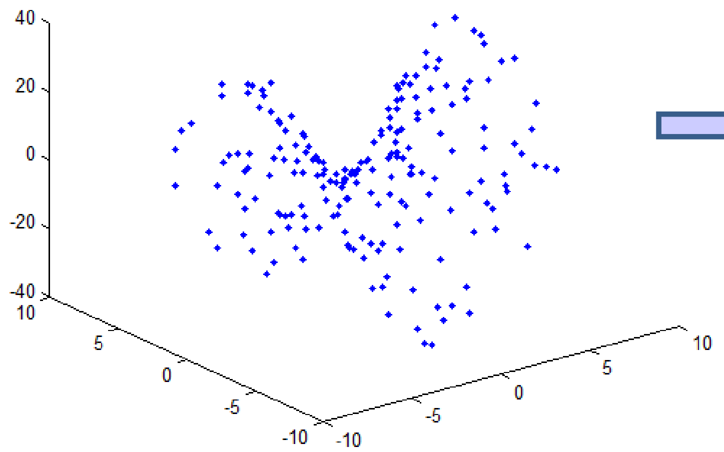
A 2-dimension function Example 2

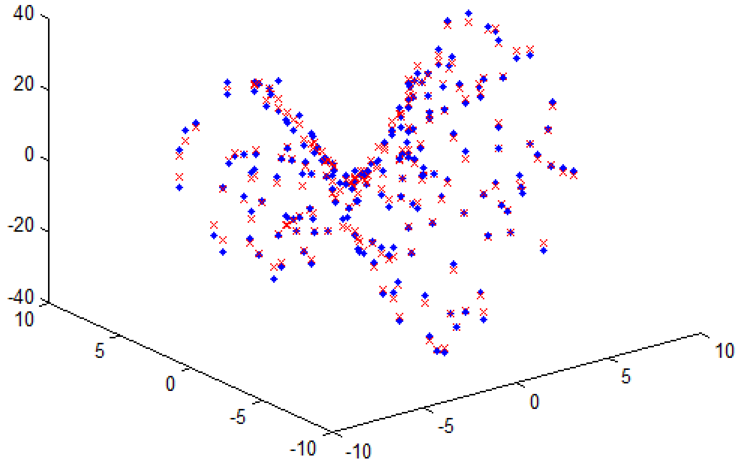
$$f(x_1, x_2) = x_1^2 - x_2^2$$

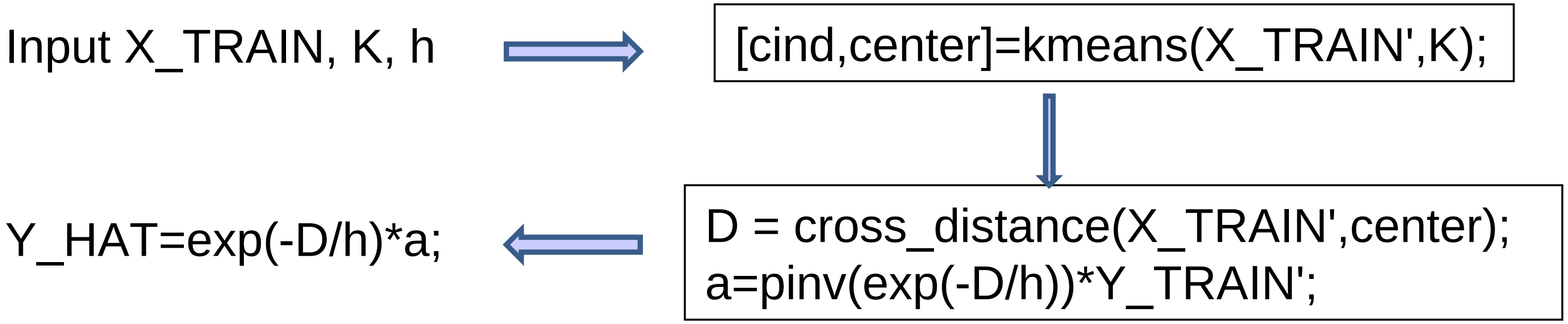
```
fstr=input('input a 2D function: x1.^2+x2.^2+cos(x1) :','s');
fx=inline(fstr);
range=2*pi;
x1=-range:0.1:range;
x2=x1;
for i=1:length(x1)
        C(i,:)=fx(x1(i),x2);
end
mesh(x1,x2,C);
```



- K=20,h=40


training points


blue points: training points   red x-mark: predicting points


Input X_TRAIN, K, h

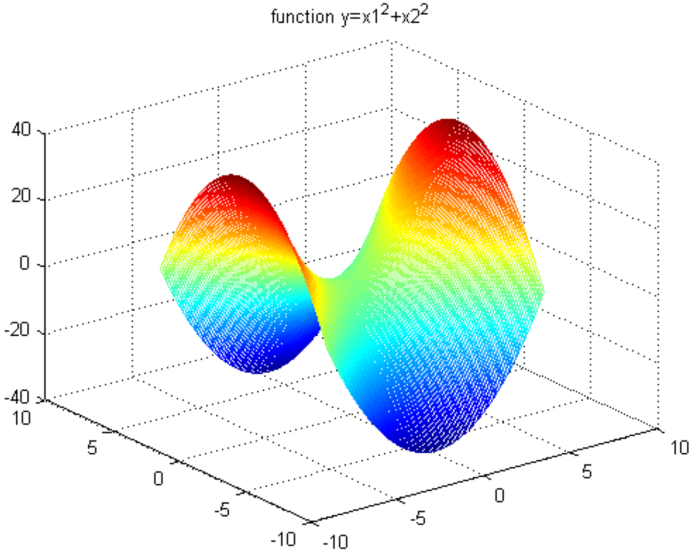
[cind,center]=kmeans(X_TRAIN',K);

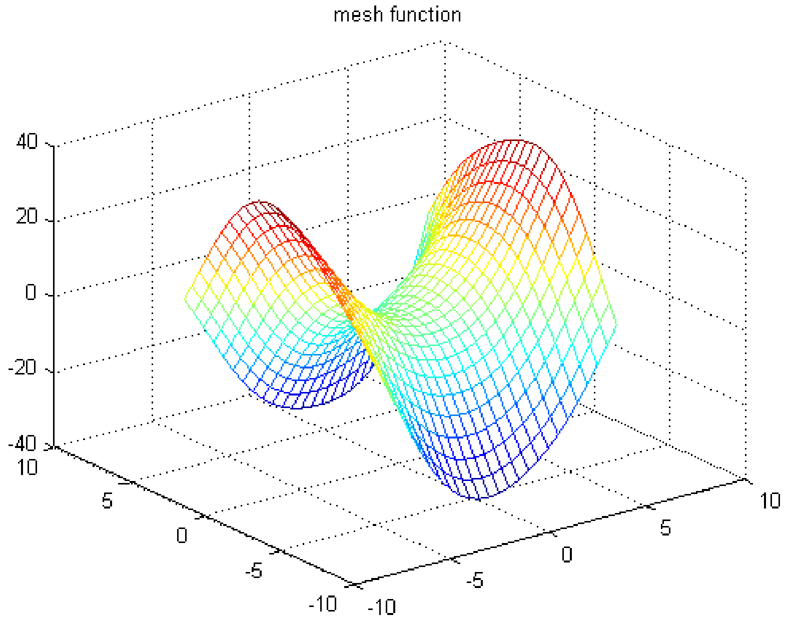
D = cross_distance(X_TRAIN',center);
a=pinv(exp(-D/h))*Y_TRAIN';

Y_HAT=exp(-D/h)*a;

- Form X_test

```
range=2*pi;
x1=-range:0.5:range;
x2=x1;
X_test=[ ];
for i=1:length(x1)
    xx = [x1(i)*ones(1,length(x2));x2];
    X_test=[X_test xx];
end
figure
plot(X_test(1,:),X_test(2,:),'.')
```





Input X_test, a, h

```
D = cross_distance(X_test',center);
```

```
figure
Y_test=exp(-D/h)*a;
CC=reshape(Y_test,length(x1),length(x1));
mesh(x1,x2,CC);
```