Exercise
MathSoft
Applied Mathematics, NDHU
Due to 06/18/2009
Problems 1-4 need to be solved.

1. Describe the Hanoi Tower Problem (HTP). Write a matlab function to solve the HTP problem.
2. Let X be an Nx2 matrix, Y be an Mx2 matrix and D be an NxM matrix, where D(i,j)= sqrt((X(i,;)-Y(j,:)).^2) measures the distance between X(i,:) and Y(j,:). Write a matlab function to determine D for given X and Y.
   A. Use nested for-loops to write your matlab function.
   B. Use vector codes to write your matlab function.
3. Draw a flow chart to illustrate the K-means algorithm for seeking centers of K clusters of given data. Explain the flow chart briefly.
4. Write a matlab function to implement the K-mean algorithm. Give two examples to verify your matlab function.
5. Download plot_2D.m and use it to plot the following 2D functions
   A. f(x)=tanh(x1+x2)-tanh(x1-x2)
   B. f(x)= 0.5*x1+0.3*x2+1
   C. f(x)=sin(x1-x2)+cos(x1+x2)
6. A hyper-plane can be expressed by,

$$y = a_1 x_1 + a_2 x_2 + ... + a_d x_d + b \quad \text{eq(1).}$$

   Implement the following matlab function to form a sample from the hyper-plane.
   A. Head: function [x y] =HPsampling(a,b,n)
   B. Body
      d = length(a)
      Use rand to generate an nxd matrix and store it to matrix x.
      Rescale elements in x within [-1,1]
      Substitute each row of x to eq (1) to form a vector y.
      y = y + rand(n,1)*0.02-0.01

7. Let x be a nxd matrix, y be a nx1 vector and **a**=[a_1,…,a_d]. Write a matlab function to calculate the following mean square error,

$$\mathrm{E}(\mathbf{a},b)=\frac{1}{n}\sum_{i=1}^{n}[y_i-(a_1x_{i1}+a_2x_{i2}+...+a_dx_{id}+b)]^2$$

   A. Head: function e = mse(x,y,a,b)

   B. Body

      Let n denote the row number of x

      Substitute each row of x to eq (1) to form vector y_hat

      e = mean(y-y_hat)

8. Let x be a nxd matrix, y be a nx1 vector and d=3. Solving eq (2) can be shown to minimize E(**a**,b) with respect to **a** and b.

$$\begin{pmatrix} \sum_i x_{i1}x_{i1} & \sum_i x_{i2}x_{i1} & \sum_i x_{i3}x_{i1} & \sum_i x_{i1} \\ \sum_i x_{i1}x_{i2} & \sum_i x_{i2}x_{i2} & \sum_i x_{i3}x_{i2} & \sum_i x_{i2} \\ \sum_i x_{i1}x_{i3} & \sum_i x_{i2}x_{i3} & \sum_i x_{i3}x_{i3} & \sum_i x_{i3} \\ \sum_i x_{i1} & \sum_i x_{i2} & \sum_i x_{i3} & N \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ b \end{pmatrix} = \begin{pmatrix} \sum_i y_i x_{i1} \\ \sum_i y_i x_{i2} \\ \sum_i y_i x_{i3} \\ \sum_i y_i \end{pmatrix} \quad eq(2)$$

   Let C and e respectively denote the left-product matrix and the right-hand-side vector of eq (2). Implement the following function to determine C and e.

   A. Head: function [C,e]=CE(x,y)

   B. Body:

      CC = x'*x

      C1 = sum(x,2);

      C2 = [C1' length(y)];

      Form the first d rows of C by combining CC, C1

      Append C2

      e = [x'*y sum(y)];

9. Write a matlab script to implement the following procedure for hyper-plane fitting. Let d = 2, n = 300, a=[1 -2] and b=3.

   i. Use plot_2D to plot the hyper-plane defined by $y=a_1x_1+a_2x_2+b$

   ii. Use HPsampling to generate x and y for given a,b and n.

   iii. Call function CE.m to determine C and e

   iv. Calculate p = inv(C)*e

   v. Set a_hat = p(1:d) and b_hat = p(d+1). Display a_hat and b_hat.

   vi. Call mse(x,y,a_hat,b_hat) to calculate the mean square error.