Mathematical Software Design
Midterm problem sets

April 21, 2009

I Write matlab functions to accomplish the following tasks (ex1)

   (a) Calculate the area of a circle for a given radius.

   (b) Calculate the area of an ellipse for given half lengths of semimajor axis and semiminor axis.

   (c) Swap contents of two variables.

   (d) Calculate the area of a circle if one number is given and the area of an ellipse if two numbers are given.

   (e) Calculate the following function,

$$f(x) = \tanh(x).$$

   and plot $f(x)$ for $x \in [-5, 5]$.

   (f) Plot 500 points that are uniformly distributed within $[-1\ 1] \times [-1\ 1]$.

   (g) Evaluate the following piecewise linear function and plot $f(x)$ with $x \in [-5\ 5]$

$$
\begin{aligned}
f(x) &= 2, & \text{if } x > 3 \\
&= x - 1, & \text{if } 3 \geq x > 0 \\
&= -x, & \text{if } 0 \geq x.
\end{aligned}
$$

II Write matlab scripts to accomplish the following tasks. (ex2)

   (a) Let A denote a gray facial image.

      1. Rotate image A 90^0 clockwise and counter-clockwise respectively and display the rotated images.

      2. Display vertical and horizontal mirrors of image A.

      3. Display the right-upper quarter of image A.

   (b) Let S be a vector that stores recorded sound data.

      1. Plot recorded sound data in a figure.

      2. Plot the reverse of recorded sound data in a figure.

      3. Repeat the data three times and plot the extended sound data in a figure.

   (c) Plot the following functions.

      1. A hyper-tangent function within [-5,5].

      2. A noisy hyper-tangent function within [-5,5].

      3. $cos(x)$ and $sin(x)$ in the same figure within $[-2\pi, 2\pi]$.

      4. $cos(x)$ and $sin(x)$ in two subplots within $[-2\pi, 2\pi]$.

III Write matlab scripts to accomplish the following tasks (ex3)

(a) Line fitting to paired data

1. Create paired data, denoted by $S = \{(x_i, y_i)\}_{i=1}^N$, using

$$y_i = ax_i + b + n_i,$$

where a and b are constants and all n_i denote noises uniformly sampled from the interval $[-0.1, 0.1]$.

2. Write a matlab function to estimate a and b for given S.

3. Let $\hat{a}$ and $\hat{b}$ denote estimated parameters. Plot paired data in S and the line expressed by $y = \hat{a}x + \hat{b}$ in a figure.

(b) Use instruction 'find' to accomplish the following tasks.

1. Generate a uniform sample from [-1 1]x[-1 1]. Plot points of quadrant II in red dots and the remains in blue dots.

2. Generate a uniform sample from [-1 1]x[-1 1]. Plot points within a unitary circle in red dots.

3. Evalute the following piecewise linear function

$$
\begin{aligned}
f(x) &= 2, && \text{if } x > 3 \\
&= x - 1, && \text{if } 3 \geq x > 0 \\
&= -x, && \text{if } 0 \geq x.
\end{aligned}
$$

Your matlab function is required to deal with vector inputs. Plot $f(x)$ with x belonging [-5 5].

(c) Solve

$$
\begin{aligned}
2x + y - z &= 1 \\
-3x - 2y + 5z &= 0 \\
x + y + z &= 5
\end{aligned}
$$

(d) Give matlab instructions to generate the following matrices

$$
A = \begin{bmatrix} 1 & 7 & 4 & 10 \\ 2 & 8 & 5 & 11 \\ 3 & 9 & 6 & 12 \end{bmatrix} \cdots \begin{bmatrix} 298 \\ 299 \\ 300 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 & 3 & 1 & 2 & 3 & 1 & 2 & 3 \\ 4 & 5 & 6 & 4 & 5 & 6 & 4 & 5 & 6 \\ 7 & 8 & 9 & 7 & 8 & 9 & 7 & 8 & 9 \end{bmatrix}
$$

IV Write matlab scripts to accomplish the following tasks. (ex 4)

(a) Let $\mathbf{x} = [x_1, ..., x_N]$, $\mathbf{y} = [y_1, ..., y_N]$ and (x_i, y_i) represent a point in a plane.

1. Form $\mathbf{A}$ and $\mathbf{b}$, where

$$
\mathbf{A} = \begin{bmatrix} \sum_{i=1}^N x_i^2 & \sum_{i=1}^N x_i \\ \sum_{i=1}^N x_i & N \end{bmatrix},
$$

$$
\mathbf{b} = \begin{bmatrix} \sum_{i=1}^N x_i y_i \\ \sum_{i=1}^N y_i \end{bmatrix}.
$$

2. Find vector $\mathbf{c}$ such that $\mathbf{Ac} = \mathbf{b}$.

(b) Quadratic curve fitting to paired data

1. Let $\mathbf{x} = [x_1, ..., x_n]$, $\mathbf{y} = [y_1, ..., y_n]$, where (x_i, y_i) denotes a noisy sample from

$$y = ax^2 + bx + c + \epsilon,$$

where a, b and c are constants and ϵ denotes a uniform noise within $[-0.1, 0.1]$. Generate vectors $\mathbf{x}$ and $\mathbf{y}$.

2. Form $\mathbf{E}$ and $\mathbf{d}$, where

$$\mathbf{E} = \begin{bmatrix} \sum_{i=1}^{n} x_i^4 & \sum_{i=1}^{n} x_i^3 & \sum_{i=1}^{n} x_i^2 \\ \sum_{i=1}^{n} x_i^3 & \sum_{i=1}^{n} x_i^2 & \sum_{i=1}^{n} x_i \\ \sum_{i=1}^{n} x_i^2 & \sum_{i=1}^{n} x_i & n \end{bmatrix},$$

$$\mathbf{d} = \begin{bmatrix} \sum_{i=1}^{n} x_i^2 y_i \\ \sum_{i=1}^{n} x_i y_i \\ \sum_{i=1}^{n} y_i \end{bmatrix}.$$

3. * Find vector $\mathbf{z} = [a, b, c]^T$ such that $\mathbf{Ez} = \mathbf{d}$.

V Write matlab functions (ex 4)

(a) Let $\mathbf{x} = [x_1, x_2]^T$ and $F(\mathbf{x}) = [f_1(x_1, x_2), f_2(x_1, x_2)]^T$, where

$$\begin{aligned} f_1(x_1, x_2) &= 3x_1^2 + x_2^2 - 16 \\ f_2(x_1, x_2) &= x_1^2 - x_2^2 + 12, \end{aligned}$$

Determine $\mathbf{F}(\mathbf{x})$ for given $\mathbf{x}$.

(b) Calculate $n!$ for given n.

(c) Let $f(x) = 2x^2 + x - 10 - \cos(x) = 0$.

1. Use fzero to find a root of $f(x)$.
2. Find multiple distinct roots of $f(x)$.

(d) Determine S_N for given N, where

$$S_N = \sum_{n=1}^{N} \left(\left\lfloor \frac{n^2}{5} \right\rfloor + \left\lceil \frac{2 * n}{3} \right\rceil \right).$$

VI Write matlab scripts to accomplish the following tasks. (ex 5)

(a) Let $\mathbf{p} = [p_1, p_2, p_3, p_4]$ be a vector whose four elements respectively denote probabilities of generating characters 'A', 'T','C', and 'G'.

1. Use a for-loop to form $\mathbf{c} = [c_1, c_2, c_3, c_4]$, where

$$c_i = \sum_{j=1}^{i} p_j.$$

2. Use rand to generate a number r within [0,1] and Use the following rule to generate a character,

$$
\begin{aligned}
R(r) &= \text{'A', } if \quad\quad r \leq c_1 \\
&= \text{'T', } if \ c_1 < r \leq c_2 \\
&= \text{'C', } if \ c_2 < r \leq c_3 \\
&= \text{'G', } if \ c_3 < r \leq c_4
\end{aligned}
$$

3. Use a for-loop to generate a string S whose characters are generated by rule R.

4. Let $\mathbf{h} = [h_1, h_2, h_3, h_4]$ and its four elements respectively denote occurrences of 'A', 'T','C', and 'G' in a string S. Determine $\mathbf{h}$ for given S.

(b) Let $fstr$ denote a string that specifies a one-dimensional function $f(x)$.

1. Plot $f(x)$ with $x \in [-5, 5]$.
2. * Plot $f'(x)$ with $x \in [-5, 5]$.

(c) Let A and B denote two N-by-N matrices and $C = AB$. Use nested for-loops to determine C for given A and B. The inner statement of the nested for-loops could be

$$C(i, j) = A(i, :) * B(:, j).$$

(d) Let $\mathbf{x} = [x_1, x_2]$ and $g(\mathbf{x}) = (x_1^2 + x_2^2 - 13)^2$. Determine $g(\mathbf{x})$ for given $\mathbf{x}$.

(e) Let $\mathbf{x} = [x_1, x_2]$. $V(\mathbf{x}) = 1$ if

$$
\begin{aligned}
-2 &\leq x_1 \leq 10 \\
0 &\leq x_2 \leq 10 \\
x_2 &\leq x_1
\end{aligned}
$$

and $V(\mathbf{x}) = 0$ otherwise. Determine $V(\mathbf{x})$ for given $\mathbf{x}$.