Exponential generating function

$$E(x) = \sum_{n=0}^{\infty} \frac{x^n}{n!}.$$

Exponential generating function

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

the exponential generating function for the number of binary strings of length n is

$$e^{2x} = \sum_{n=0}^{\infty} \frac{(2x)^n}{n!} = \sum_{n=0}^{\infty} 2^n \frac{x^n}{n!}.$$

$$e^{x} = \sum_{n=0}^{\infty} \frac{x^{n}}{n!}$$

$$e^{-x} = \sum_{n=0}^{\infty} \frac{e^{-x}}{n!}$$

$+$

$$e^{x} + e^{-x} = 2\left(1 + \frac{x^{2}}{2!} + \frac{x^{4}}{4!} + \cdots\right)$$

$$\frac{1}{2}\left(e^{x} + e^{-x}\right) =$$

$$1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \cdots = \sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!}.$$

Suppose we wish to find the number of ternary strings which the number of 0s is even

For 1s and 2s, since we may have any number of each of them, we introduce a factor of e^x for each

$$\frac{e^x + e^{-x}}{2} e^x e^x = \frac{e^{3x} + e^x}{2} = \frac{1}{2}\left(\sum_{n=0}^{\infty} \frac{3^n x^n}{n!} + \sum_{n=0}^{\infty} \frac{x^n}{n!}\right).$$

$$(3^n + 1)/2$$

How many ternary strings of length n have at least one 0 and at least one 1? To ensure that a symbol appears at least once, we need the following exponential generating function

$$x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots = \sum_{n=1}^{\infty} \frac{x^n}{n!}.$$

$$\sum_{n=1}^{\infty} x^n / n! = e^x - 1$$

At least one 1 and one 0

$$(e^x - 1)(e^x - 1)e^x = e^{3x} - 2e^{2x} + e^x$$

$$= \sum_{n=0}^{\infty} \frac{3^n x^n}{n!} - 2\sum_{n=0}^{\infty} \frac{2^n x^n}{n!} + \sum_{n=0}^{\infty} \frac{x^n}{n!}.$$

ANS

$$3^n - 2 \cdot 2^n + 1$$