

# Topics

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# Discrete Mathematics

Elementary and Beyond

With 95 Illustrations

4

Fibonacci Numbers

## 4 Fibonacci Numbers

- 4.1 Fibonacci's Exercise . . . . .
- 4.2 Lots of Identities . . . . .
- 4.3 A Formula for the Fibonacci Numbers .

In the thirteenth century, the Italian mathematician Leonardo Fibonacci studied the following (not too realistic) question:



Leonardo Fibonacci

*A farmer raises rabbits. Each rabbit gives birth to one rabbit when it turns 2 months old, and then to one rabbit each month thereafter. Rabbits never die, and we ignore male rabbits. How many rabbits will the farmer have in the  $n$ th month if he starts with one newborn rabbit?*

## **Graphs**

- 7.1 Even and Odd Degrees . . . . .
- 7.2 Paths, Cycles, and Connectivity . . . . .
- 7.3 Eulerian Walks and Hamiltonian Cycles

## 8 Trees

- 8.1 How to Define Trees . . . . .
- 8.2 How to Grow Trees . . . . .
- 8.3 How to Count Trees? . . . . .
- 8.4 How to Store Trees . . . . .
- 8.5 The Number of Unlabeled Trees

<b>9</b>	<b>Finding the Optimum</b>	
9.1	Finding the Best Tree	. . . . .
9.2	The Traveling Salesman Problem	



# 10 Matchings in Graphs

- 10.1 A Dancing Problem . . . . .
- 10.2 Another matching problem . . . . .
- 10.3 The Main Theorem . . . . .
- 10.4 How to Find a Perfect Matching

## **13 Coloring Maps and Graphs**

- 13.1 Coloring Regions with Two Colors . . . . .
- 13.2 Coloring Graphs with Two Colors . . . . .
- 13.3 Coloring graphs with many colors . . . . .
- 13.4 Map Coloring and the Four Color Theorem

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Topics in Finite and Discrete Mathematics

Sheldon M. Ross

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## Chapter

3 - Probability pp. 70-96

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Sections 1-5

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Chapter

Sections 1-5

3 - Probability pp. 70-96

## **4. Mathematics of Finance**      Sections 1-5

Chapter

5 - Graphs and Trees pp. 124-149      Sections 1-5

Chapter

6 - Directed Graphs pp. 150-179      Sections 1-4

Chapter

7 - Linear Programming pp. 180-202      Sections 1-4

Chapter

8 - Sorting and Searching pp. 203-219      Sections 1,2-6

# 4. Mathematics of Finance

Sections 1-5

## 4.1 Interest Rates

If you borrow the amount  $P$  (called the principal) which must be repaid at time  $T$  along with simple interest at rate  $r$  per time  $T$ , then the amount to be repaid at time  $T$  is

$$P + rP = (1 + r)P.$$

$$P \lim_{n \rightarrow \infty} (1 + r/n)^n = Pe^r,$$

where  $e$ , the base of the natural logarithm, is defined by

$$e = \lim_{n \rightarrow \infty} (1 + 1/n)^n$$

and is approximately given by  $e \approx 2.71828 \dots$

## Chapter

5 - Graphs and Trees pp. 124-149

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Sections 1-5





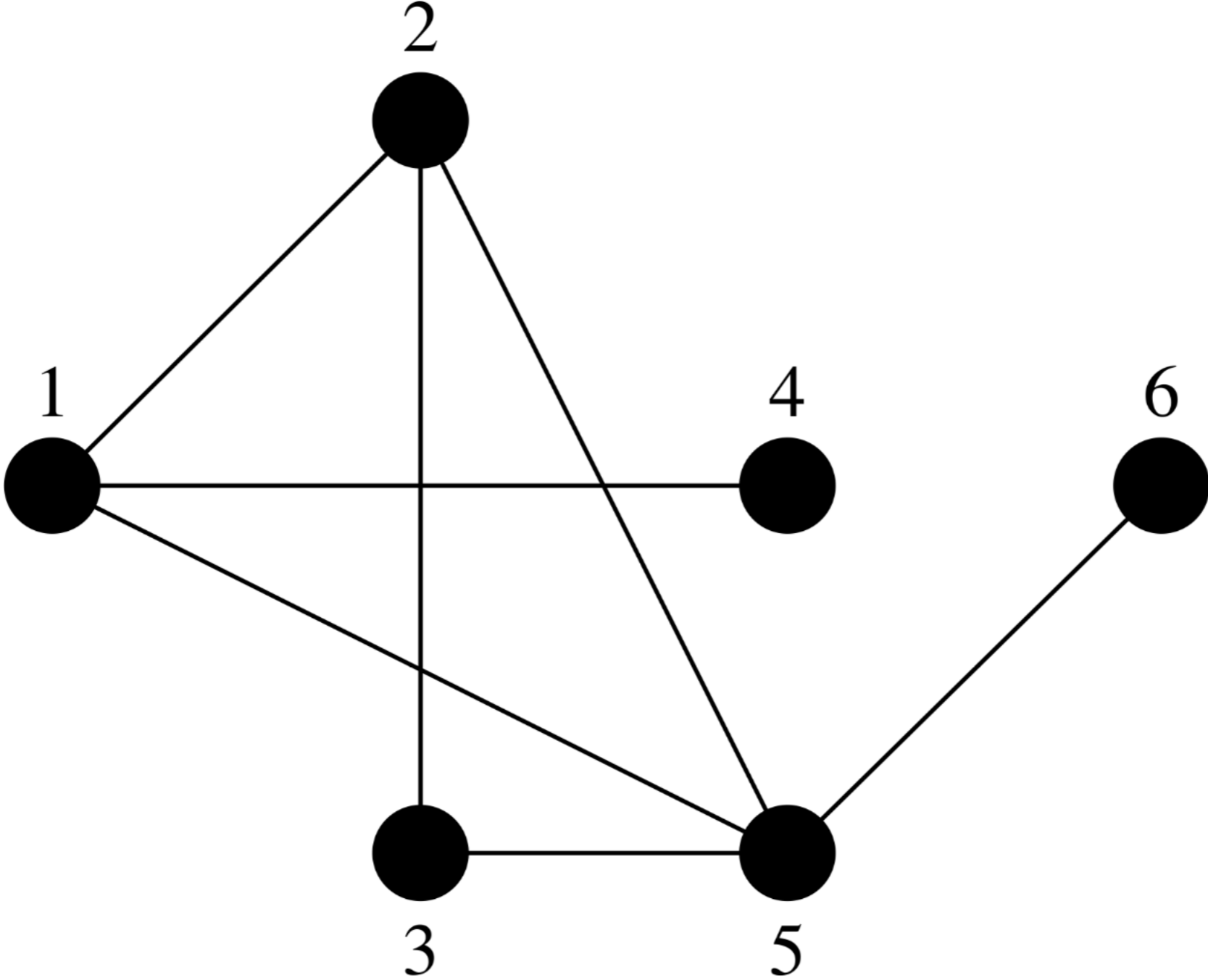


Figure 5.1: A Graph

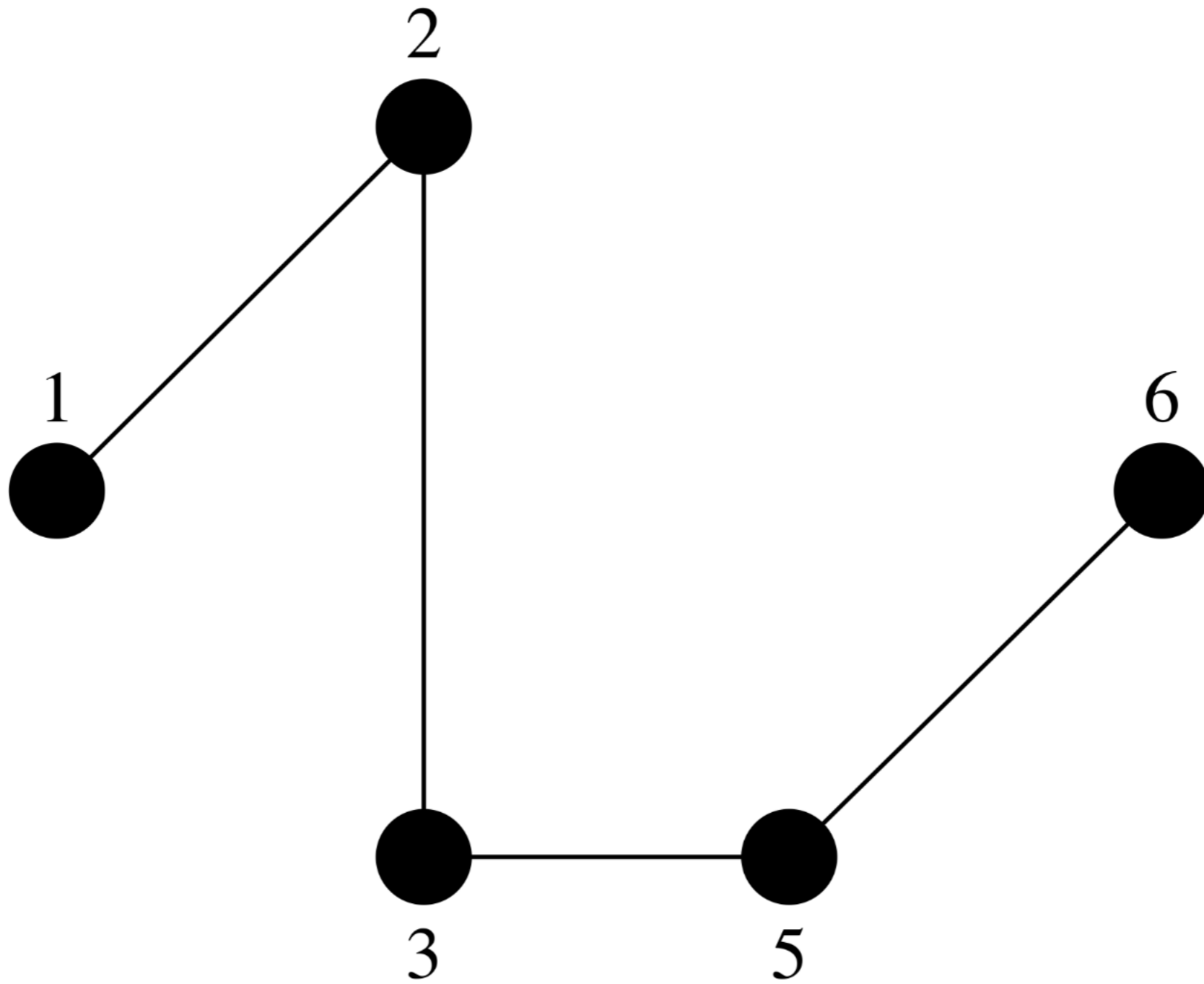


Figure 5.2: A Path from 1 to 6: 1, 2, 3, 5, 6

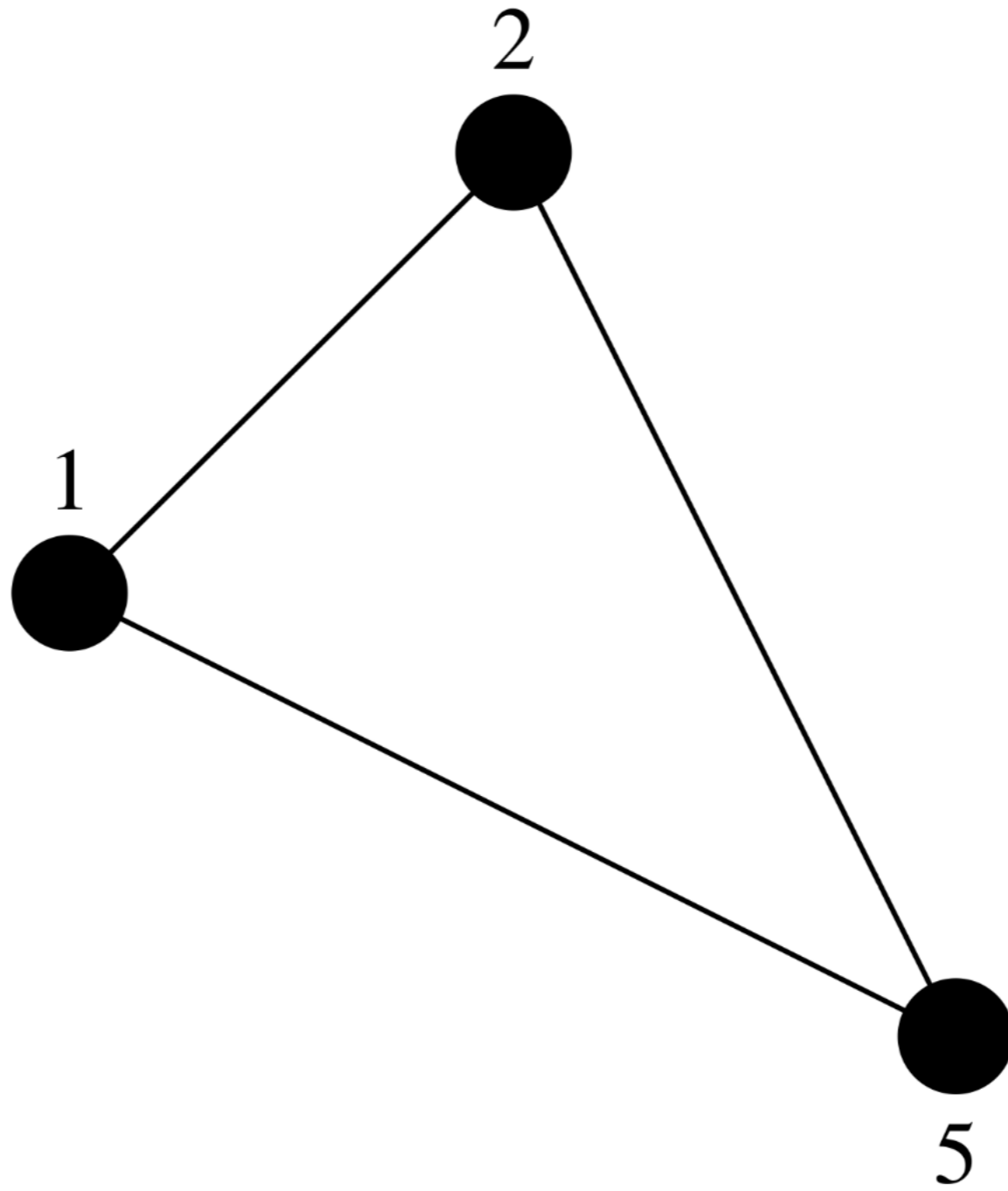


Figure 5.3: A Cycle: 1, 2, 5, 1

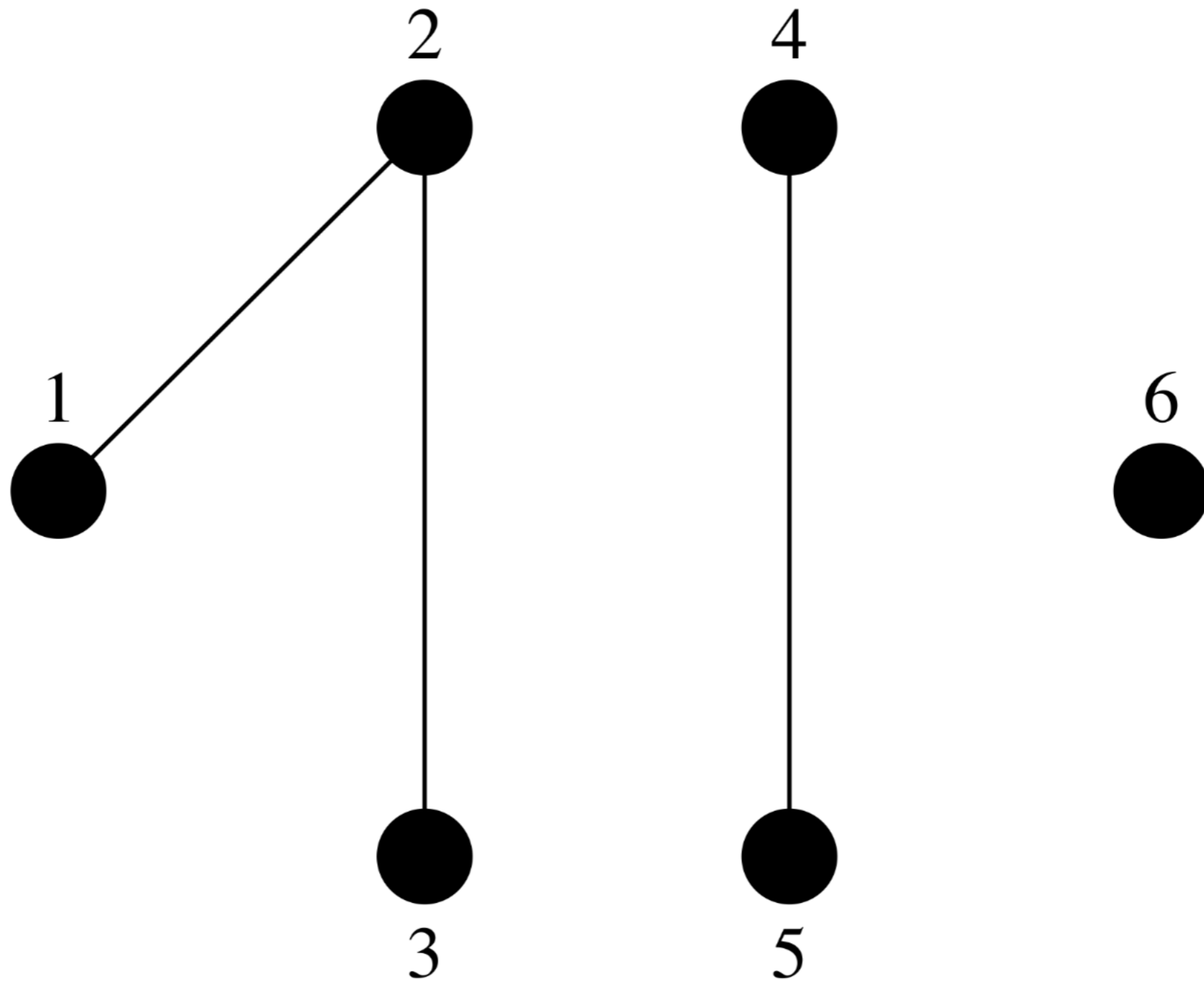


Figure 5.4: A Graph with Three Components

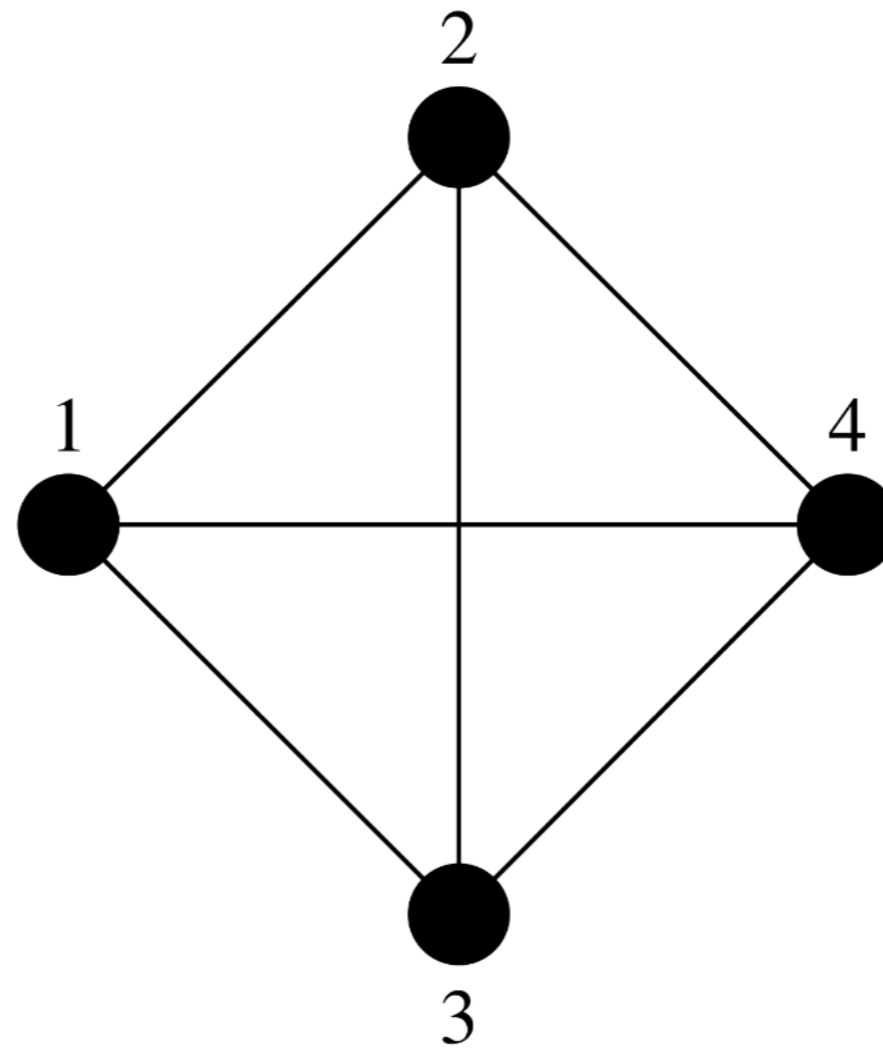


Figure 5.5: The Complete Graph on Four Vertices

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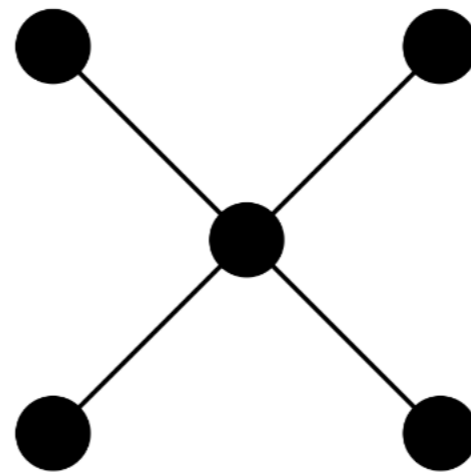
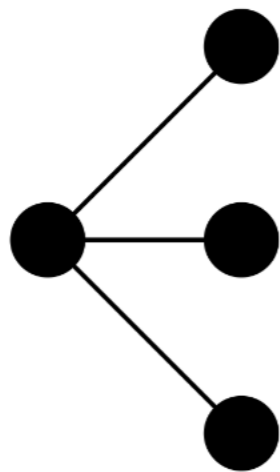
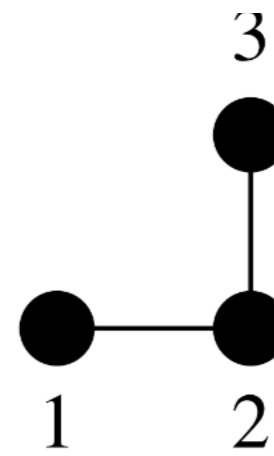
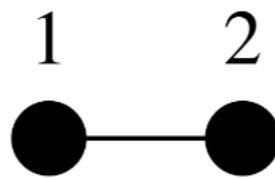


Figure 5.6: Trees





# Chapter

## 6 - Directed Graphs pp. 150-179

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Sections 1-4



*The Maximum Flow Problem*

151

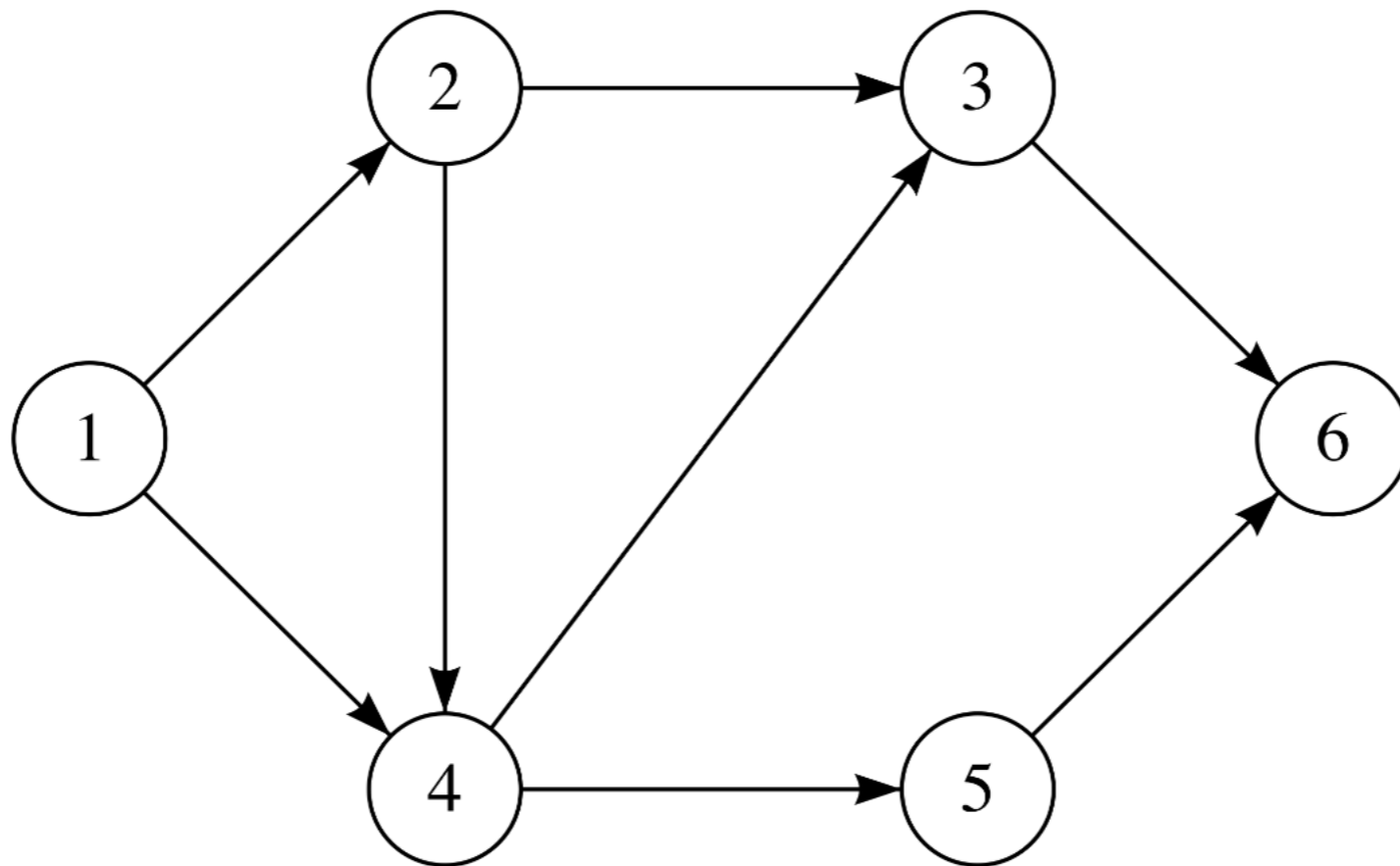


Figure 6.1: Directed Graph with

$$\mathcal{A} = \{(1, 2), (1, 4), (2, 3), (2, 4), (3, 6), (4, 3), (4, 5), (5, 6)\}$$



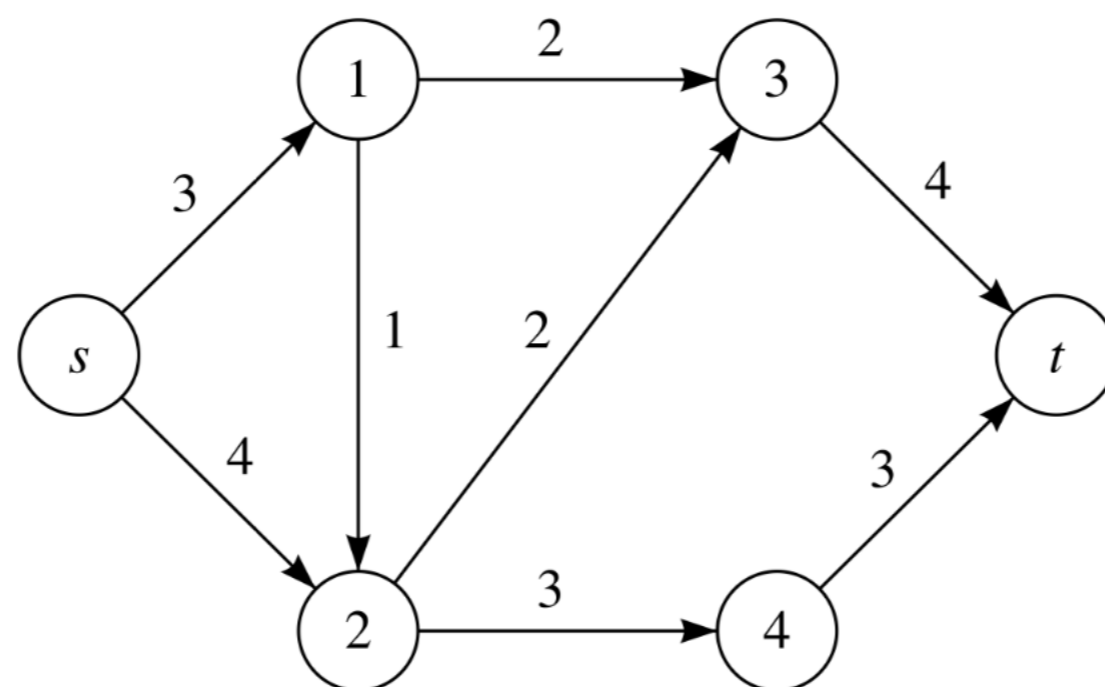
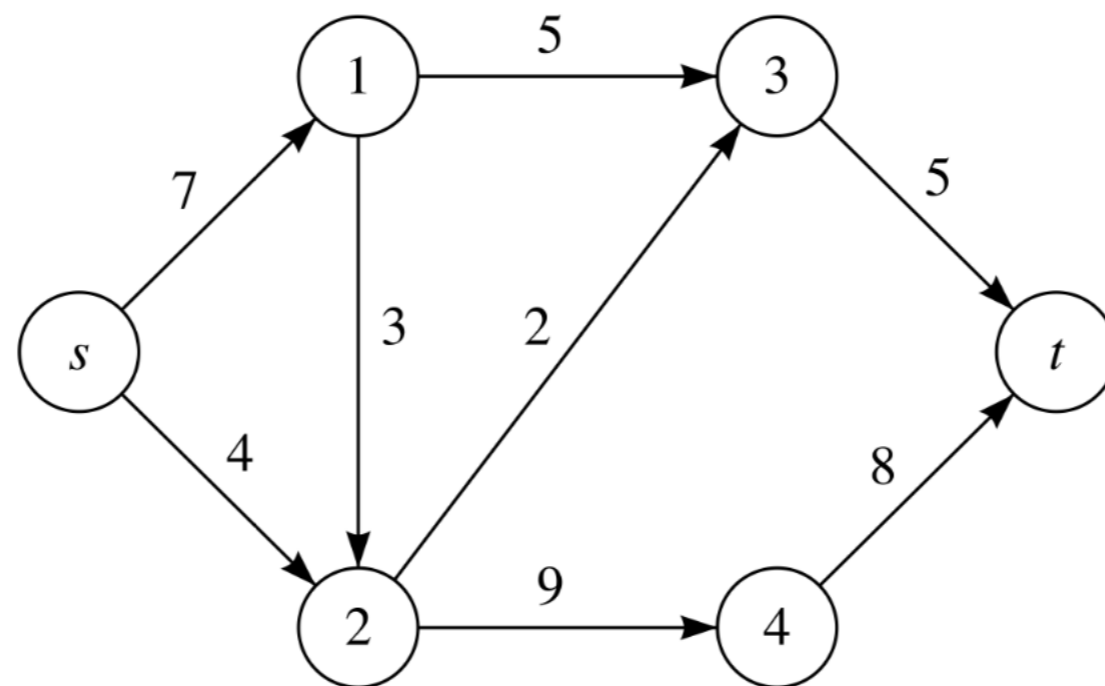


Figure 6.2: Edge Capacities and a Feasible Flow



**Theorem 6.2.1** (Max-Flow Min-Cut Theorem)

$$\max_f v(f) = \min c(X, \bar{X}).$$

Shortest Path in Digraphs 171

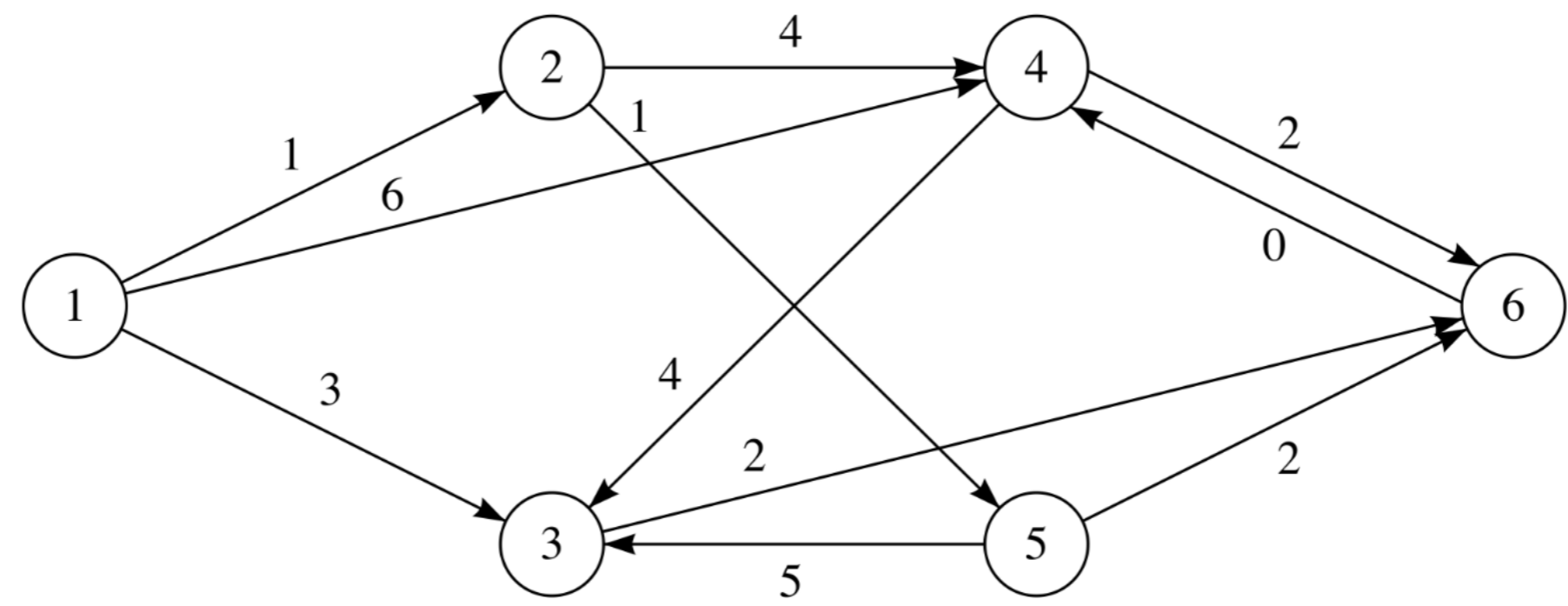


Figure 6.15: A Directed Graph with Costs

For instance, in the directed graph depicted in Figure 6.15, interpreting the numbers on the edge distances yields that the length of the path from vertex 1 to vertex 6 is

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## Chapter

7 - Linear Programming pp. 180-202

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Sections 1-4



maximize  $1 + x_1 + x_2 + x_3$

*subject to*

$$1 - x_1 \geq 0,$$

$$1 - 3x_1 - x_2 \geq 0,$$

$$1 + x_1 - 3x_2 - x_3 \geq 0,$$

$$1 + x_1 + x_2 - 3x_3 \geq 0;$$

$$x_1 \geq 0, \quad x_2 \geq 0, \quad x_3 \geq 0.$$



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## Chapter

8 - Sorting and Searching pp. 203-219

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Sections 1,2-6



5 3 8 7 0 9 6 4 1,

then (with the bar indicating the value that is to be compared with its immediate follower) the successive orderings in the first pass are as follows:

3  $\bar{5}$  8 7 0 9 6 4 1,

3 5  $\bar{8}$  7 0 9 6 4 1,

3 5 7  $\bar{8}$  0 9 6 4 1,

3 5 7 0  $\bar{8}$  9 6 4 1,

3 5 7 0 8  $\bar{9}$  6 4 1,

3 5 7 0 8 6  $\bar{9}$  4 1,

3 5 7 0 8 6 4  $\bar{9}$  1,

3 5 7 0 8 9 4 1 9.