

Discrete Math 2018

problem set 5

1. State and explain the strategy of proving $A \rightarrow B$ for each of the following cases.

- A. $B = \sim Q$ and Q is given.
- B. $B = \exists x P(x)$ and A is given.
- C. $B = \sim \forall x P(x)$ and A is given.
- D. $B = C \vee D$ and $A \wedge \sim C$ is given.
- E. $B = \forall x \in \mathbb{N} P(x)$ and A is given.

2. Let E_x denote the predicate that x is even.

$$A_1 : \forall x: E_x \leftrightarrow (x=0 \vee (\exists y: E_y \wedge x=SSy))$$

$$A_2 : \forall x: 0=Sx.$$

$$A_3 : \forall x \forall y: Sx=Sy \rightarrow x=y.$$

A. Express the second version of inference rule of implication elimination ($\rightarrow E_2$).

B. Apply ($\rightarrow E_2$) to prove $\Gamma \vdash \neg E(S0)$ given $\Gamma \vdash E(S0) \rightarrow Q$ for some Q with $\Gamma \vdash \neg Q$;

C. State the inference rule of $\forall E$.

D. Derive the result of applying rule $\forall E$ to A_1 .

E. State the rule of ($\rightarrow E$).

F. Apply rule ($\rightarrow E$) to statements

$$\Gamma \vdash E(S0)$$

$$\Gamma \vdash E(S0) \rightarrow (S0=0) \vee \exists y : (E_y \wedge S0 = SSy)$$

G. State the inference rule of $\vee E_1$.

H. Derive the result of applying it to statements of

$$\Gamma, E(S0) \vdash \neg(S0=0)$$

$$\Gamma, E(S0) \vdash (S0=0) \vee \exists y : (E_y \wedge S0 = SSy)$$