Ludwig Boltzmann

- Ludwig Boltzmann - Wikipedia, the free encyclopedia

Boltzmann distribution

- Boltzmann distribution - Wikipedia, the free encyclopedia

Maxwell-Boltzmann distribution

- 馬克斯威-波茲曼分佈

- Maxwell_Boltzmann_statistics

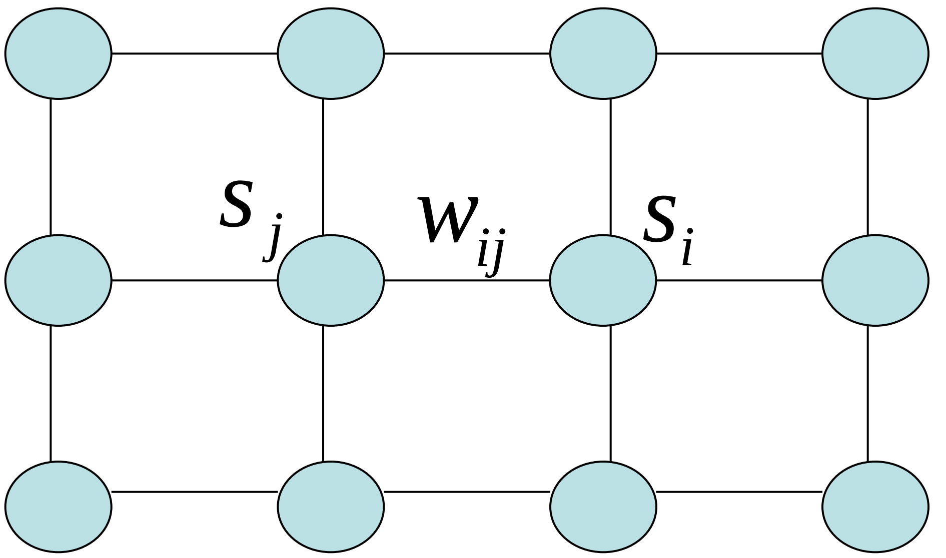
Fermi_Dirac distribution

- Fermi_Dirac_statistics

Logistic differential equation

- Logistic function - Wikipedia, the free encyclopedia

- Sigmoid differential equation



$$s_i \in \{-,+\}$$

$$v_i = \langle s_i \rangle \in [-,+]$$

$$s_i \in \{0,1\}$$

$$u_i = \langle s_i \rangle \in [0,1]$$

$$h_i = \sum_{j \in NB_i} w_{ij} v_j$$

logistic differential equation $\dfrac{du_i}{dh_i} = \beta u_i (1 - u_i)$

$$u_i = f(h_i) = \frac{1}{\exp(-\beta h_i) + 1}$$

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logistic differential equation $\dfrac{du_i}{dh_i} = \beta u_i (1 - u_i)$

$$u_i = f(h_i) = \frac{1}{\exp(-\beta h_i) + 1}$$

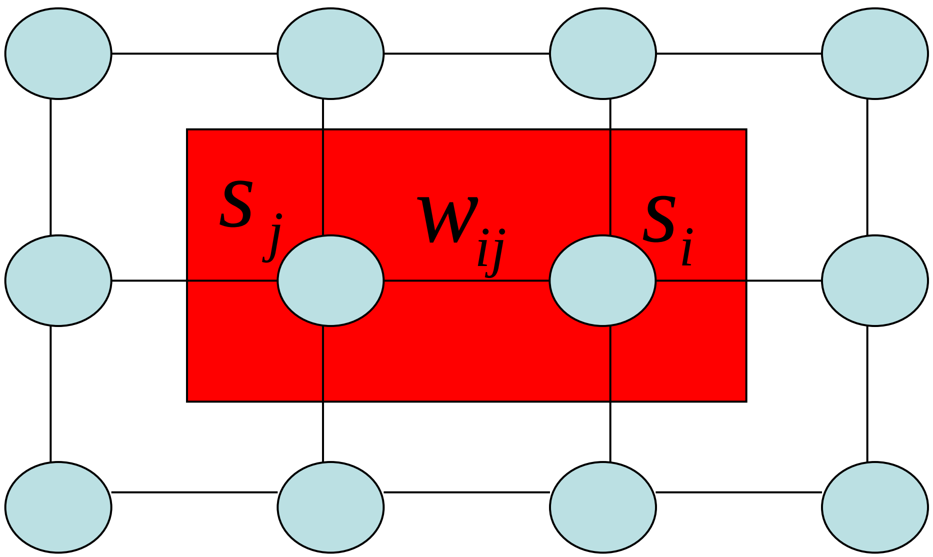
$$v_i = 2u_i - 1,$$

$$\frac{dv_i}{dh_i} = 2\frac{du_i}{dh_i} = 2\beta\left(\frac{1+v_i}{2}\right)\left(\frac{1-v_i}{2}\right)$$

$$= \frac{1}{2}\beta(1+v_i)(1-v_i)$$

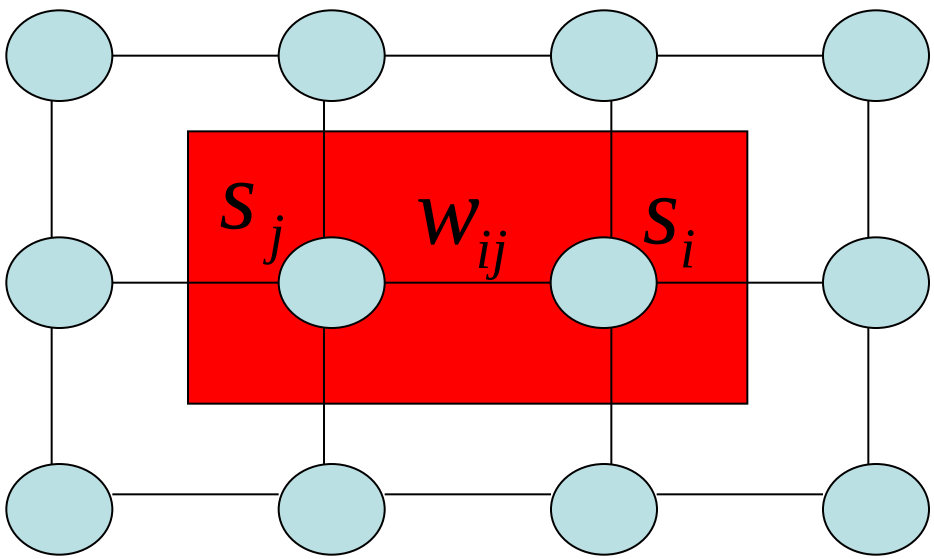
$$v_i = \tanh(\beta h_i)$$

- couple expectation



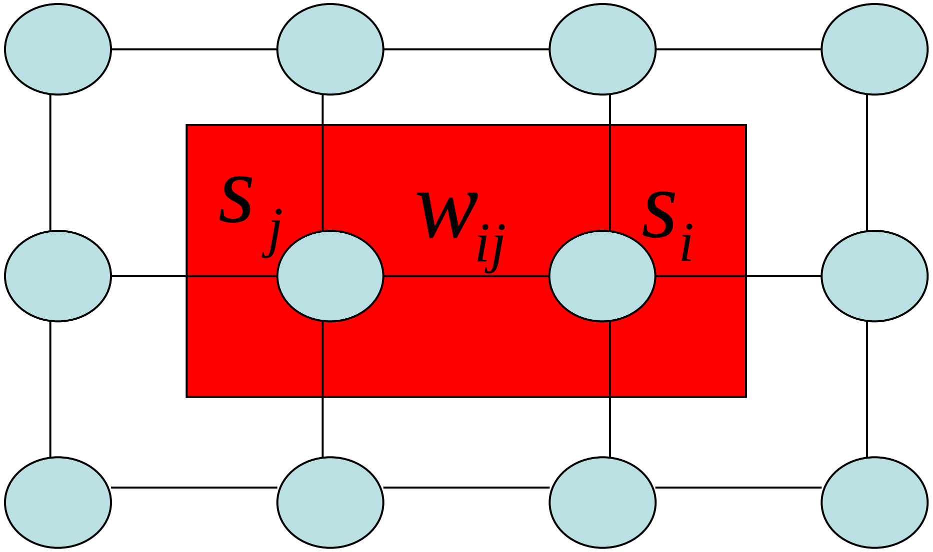
$$s_i \in \{-,+\}$$

$$q_{ij} = s_i s_j \in \{-,+\}$$



$$h_i = \sum_{\substack{k \in NB(i) \\ k \neq i}} w_{ik} v_k$$

$$h_j = \sum_{\substack{k \in NB(j) \\ k \neq j}} w_{jk} v_k$$

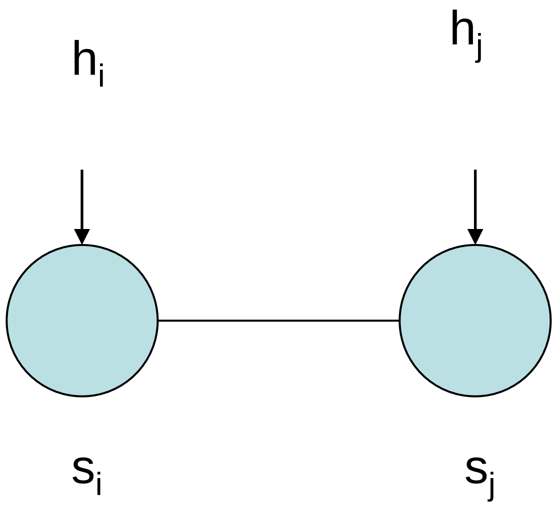


$$s_i = +, s_j = + \qquad \varepsilon_{++} = h_i + h_j + w_{ij}$$

$$s_i = +, s_j = - \qquad \varepsilon_{+-} = h_i - h_j - w_{ij}$$

$$s_i = -, s_j = + \qquad \varepsilon_{-+} = -h_i + h_j - w_{ij}$$

$$s_i = -, s_j = - \qquad \varepsilon_{--} = -h_i - h_j + w_{ij}$$



$$\Pr(s_i = +, s_j = +) \propto \exp(-\beta\varepsilon_{++})$$

$$\Pr(s_i = +, s_j = -) \propto \exp(-\beta\varepsilon_{+-})$$

$$\Pr(s_i = -, s_j = +) \propto \exp(-\beta\varepsilon_{-+})$$

$$\Pr(s_i = -, s_j = -) \propto \exp(-\beta\varepsilon_{++})$$

$$\Pr(s_i = +, s_j = +) \propto \exp(-\beta\varepsilon_{++})$$

$$\Pr(s_i = +, s_j = -) \propto \exp(-\beta\varepsilon_{+-})$$

$$\Pr(s_i = -, s_j = +) \propto \exp(-\beta\varepsilon_{-+})$$

$$\Pr(s_i = -, s_j = -) \propto \exp(-\beta\varepsilon_{++})$$

$$\Pr(q_{ij} = 1) = \Pr(s_i = +, s_j = +) + \Pr(s_i = -, s_j = -)$$

$$= \frac{\exp(-\beta\varepsilon_{++}) + \exp(-\beta\varepsilon_{--})}{\exp(-\beta\varepsilon_{++}) + \exp(-\beta\varepsilon_{+-}) + \exp(-\beta\varepsilon_{-+}) + \exp(-\beta\varepsilon_{--})}$$

$$\Pr(q_{ij} = -1) = \Pr(s_i = +, s_j = -) + \Pr(s_i = -, s_j = +)$$

$$= \frac{\exp(-\beta\varepsilon_{+-}) + \exp(-\beta\varepsilon_{-+})}{\exp(-\beta\varepsilon_{++}) + \exp(-\beta\varepsilon_{+-}) + \exp(-\beta\varepsilon_{-+}) + \exp(-\beta\varepsilon_{--})}$$

$$\Pr(q_{ij}=1)=\Pr(s_i=+,s_j=+)+\Pr(s_i=-,s_j=-)$$

$$=\frac{\exp(-\beta\varepsilon_{++})+\exp(-\beta\varepsilon_{--})}{\exp(-\beta\varepsilon_{++})+\exp(-\beta\varepsilon_{+-})+\exp(-\beta\varepsilon_{-+})+\exp(-\beta\varepsilon_{--})}$$

$$\Pr(q_{ij}=-1)=\Pr(s_i=+,s_j=-)+\Pr(s_i=-,s_j=+)$$

$$=\frac{\exp(-\beta\varepsilon_{+-})+\exp(-\beta\varepsilon_{-+})}{\exp(-\beta\varepsilon_{++})+\exp(-\beta\varepsilon_{+-})+\exp(-\beta\varepsilon_{-+})+\exp(-\beta\varepsilon_{--})}$$

$$\left\langle q_{ij}\right\rangle=\Pr(q_{ij}=1)-\Pr(q_{ij}=-1)$$

$$=\frac{\exp(-\beta\varepsilon_{++})-\exp(-\beta\varepsilon_{+-})-\exp(-\beta\varepsilon_{-+})+\exp(-\beta\varepsilon_{--})}{\exp(-\beta\varepsilon_{++})+\exp(-\beta\varepsilon_{+-})+\exp(-\beta\varepsilon_{-+})+\exp(-\beta\varepsilon_{--})}$$

- $$\mathrm{KL_i} = \sum_{j \in NB(i)} p_{ij} \log \frac{p_{ij}}{p_i p_j}$$

  $\mathrm{Minimize\ KL_i\ to\ determine\ individual\ p_i}$

  $\mathrm{Given}\ \{\mathrm{v_i}\}, \mathrm{calculate}\ \left\langle \mathrm{s_i} s_j \right\rangle$

  $\mathrm{Given}\ \left\langle \mathrm{s_i} s_j \right\rangle, \mathrm{determine}\ \left\langle \mathrm{s_i} \right\rangle\ \mathrm{by\ minimizing\ KL_i}$

- $$\mathrm{KL_i} = \sum_{j \in NB(i)} p_i p_j \log \frac{p_i p_j}{p_{ij}}$$

  Minimize $\mathrm{KL_i}$ to determine individual $\mathrm{p_i}$

  Given $\{\mathrm{v_i}\}$, calculate $\langle \mathrm{s_i} s_j \rangle$

  Given $\langle \mathrm{s_i} s_j \rangle$, determine $\langle \mathrm{s_i} \rangle$ by minimizing $\mathrm{KL_i}$

$$\mathrm{KL_{ij}} = \sum_{s_i, s_j} p_i(s_i) p_j(s_j) \log \frac{p_i(s_i) p_j(s_j)}{p_{ij}(s_i, s_j)}$$

$$= \sum_{s_i, s_j} p_i(s_i) p_j(s_j) \{\log p_i(s_i) + \log p_j(s_j) - \log p_{ij}(s_i, s_j)\}$$

$$= \sum_{s_i} p_i(s_i) \log p_i(s_i) + \sum_{s_j} p_j(s_j) \log p_j(s_j) - \sum_{s_i, s_j} p_i(s_i) p_j(s_j) \log p_{ij}(s_i, s_j)$$

Minimize $\mathrm{KL_i}$ to determine individual $\mathrm{p_i}$

Given $\{\mathrm{v_i}\}$, calculate $\langle \mathrm{s_i} s_j \rangle$

Given $\langle \mathrm{s_i} s_j \rangle$, determine $\langle \mathrm{s_i} \rangle$ by minimizing $\mathrm{KL_{\ i}}$

Exercise I

$$y(0) = 1$$

$$\frac{dy}{dx} = x$$

The change of y with respect to x is proportional to x.

The initial value of y is one

$$y(x) = ?$$

Exercise II

$$y(0) = 1$$

$$\frac{dy}{dx} = x(1 - x)$$

The change of y with respect to x is proportional to x(1 - x).

$$y(x) = ?$$

Exercise III

$$y(0) = 1$$

$$\frac{dy}{dx} = y$$

The change of y with respect to x is proportional to y.

$$y(x) = ?$$

Exercise IV

$$y(0) = 1$$

$$\frac{dy}{dx} = y(1 - y)$$

The change of y with respect to x is proportional to y(1- y).

$$y(x) = ?$$