Markov

Markov chain &
Markov Processes

Andrey Markov - Wikipedia, the free encyclopedia

- Markov

- Markov biography

馬可夫鏈

- Wiki

馬可夫鏈，因俄羅斯數學家安德烈·馬可夫得名，是數學中具有馬可夫性質的離散時間隨機過程。該過程中，在給定當前知識或信息的情況下，只有當前的狀態用來預測將來，**過去（即當前以前的歷史狀態）對於預測將來（即當前以後的未來狀態）是無關的**。

在馬可夫鏈的每一步，系統根據機率分布，可以從一個狀態變到另一個狀態,也可以保持當前狀態。狀態的改變叫做過渡，與不同的狀態改變相關的機率叫做過渡機率。隨機漫步就是馬可夫鏈的例子。隨機漫步中每一步的狀態是在圖形中的點，每一步可以移動到任何一個相鄰的點，在這裡移動到每一個點的機率都是相同的(無論之前漫步路徑是如何的)。

Wiki (Markov chain)

- Markov chain - Wikipedia, the free encyclopedia

A "discrete-time" random process means a system which is in a certain state at each "step", with the state changing randomly between steps. The steps are often thought of as time, but they can equally well refer to physical distance or any other discrete measurement; formally, the steps are just the integers or natural numbers, and the random process is a mapping of these to states. The Markov property states that the conditional probability distribution for the system at the next step (and in fact at all future steps) *given* its current state depends only on the current state of the system, and not additionally on the state of the system at previous steps.

Since the system changes randomly, it is generally impossible to predict the exact state of the system in the future. However, the statistical properties of the system's future can be predicted. In many applications it is these statistical properties that are important.

A famous Markov chain is the so-called "drunkard's walk", a random walk on the number line where, at each step, the position may change by +1 or −1 with equal probability. From any position there are two possible transitions, to the next or previous integer. The transition probabilities depend only on the current position, not on the way the position was reached. For example, the transition probabilities from 5 to 4 and 5 to 6 are both 0.5, and all other transition probabilities from 5 are 0. These probabilities are independent of whether the system was previously in 4 or 6.

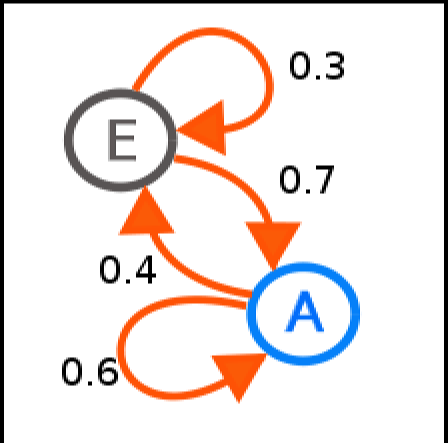
$$\Pr(x_{n+1} = 4 \mid x_n = 5) = \frac{1}{2}$$

$$\Pr(x_{n+1} = 6 \mid x_n = 5) = \frac{1}{2}$$

Another example is the dietary habits of a creature who eats only grapes, cheese or lettuce, and whose dietary habits conform to the following rules:

- It eats exactly once a day.
- If it ate cheese yesterday, it will not today.
- It will eat lettuce or grapes with equal probability.
- If it ate grapes yesterday, it will eat grapes today with probability 1/10, cheese with probability 4/10 and lettuce with probability 5/10.
- If it ate lettuce yesterday, it will not eat lettuce again today but will eat grapes with probability 4/10 or cheese with probability 6/10.

This creature's eating habits can be modeled with a Markov chain since its choice depends solely on what it ate yesterday, not what it ate two days ago or even farther in the past. One statistical property that could be calculated is the expected percentage, over a long period, of the days on which the creature will eat grapes.



E
A
0.3
0.7
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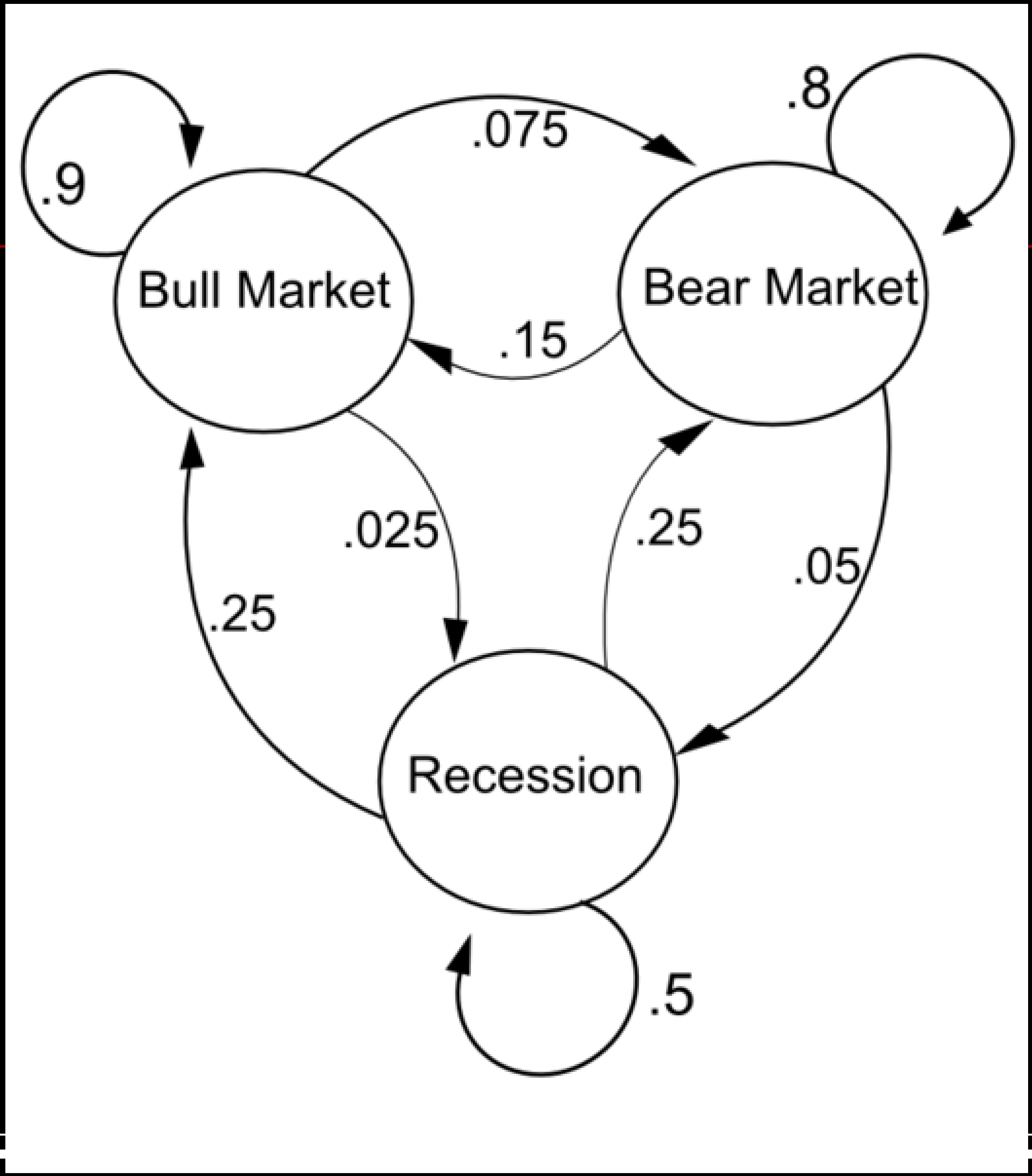

.9
.075
.8
Bull Market
Bear Market
.15
.025
.25
.05
.25
Recession
.5

Variations

- Continuous-time Markov processes have a continuous index.
- **Time-homogeneous Markov chains** (or **stationary Markov chains**) are processes where

$$\Pr(X_{n+1} = x | X_n = y) = \Pr(X_n = x | X_{n-1} = y)$$

for all *n*. The probability of the transition is independent of *n*.

- A **Markov chain of order *m*** (or a Markov chain with memory *m*) where *m* is finite, is a process satisfying

$$\begin{aligned} & \Pr(X_n = x_n | X_{n-1} = x_{n-1}, X_{n-2} = x_{n-2}, \dots, X_1 = x_1) \\ = & \Pr(X_n = x_n | X_{n-1} = x_{n-1}, X_{n-2} = x_{n-2}, \dots, X_{n-m} = x_{n-m}) \text{ for } n > m \end{aligned}$$

Markov chain

- 馬可夫鏈簡介

定義1：

一組值在 S 中的隨機變數 X_0, X_1, X_2 …… 稱為馬可夫鏈。如果它滿足以下的條件：

$$P(X_{n+1} = x_{n+1} | X_0 = x_0 \cdots X_n = x_n) = P(X_{n+1} = x_{n+1} | X_n = x_n) \qquad (1)$$

其中 $x_0, x_1, \cdots, x_{n+1}$ 皆屬於 S。

例一：

設有一賭徒甲帶了現金若干去賭場賭博。如果他每贏一次則贏一元而每輸一次也輸一元，同時贏的機率是 p（輸的機率就是 q=1-p），則 $X_0, X_1, X_2, \ldots\ldots$ 顯然是一個馬可夫鏈。（X_n 表示他賭 n 次後手中所有的資金）為什麼呢？因為僅僅需要知道 X_n 的值就足以預測 X_{n+1} 的值，另外 $X_0 X_1 \cdots X_{n-1}$ 並不能幫助我們做更好的預測。在這個例子中，

$$p(x,y)=\begin{cases} p & \text{如果} y=x+1 \\ 1-p & \text{如果} y=x-1 \\ 0 & \text{如果} y\neq x+1 \text{或} x-1 \end{cases}$$

例三：

假設有無窮多個瓶子，我們標以 $0,1,\cdots\cdots$。每一個瓶子中放不同個數球，每瓶中的球都標以 $B_0,B_1,B_2,\cdots\cdots$ 等（不同的球可以有相同的編號），假設我們從標以 0 的瓶中隨便抽一個球，則此球的編號是 B_j，我們再從第 j 個瓶中抽取一球，若此球編號是 B_k，我們再從第 k 個瓶中抽取一球，……這樣一直做下去，我們就得到一個馬可夫鏈。原因和前面幾個例子相同。事實上任一個馬可夫鏈都和這個模型等價，我們只要適當的選取每個瓶中球的個數，同時加以適當的編號，我們就可得取第一和第二個例子。

例四：

考慮一個基因由 n 個單位所合成的。在此 n 個單位中，每一個不是正常的就是突變的。現在這些細胞要分裂，分裂前基因先要完全複製一次，也就是有了 $2n$ 個單位。假設一個細胞一分為二，則此二個子細胞中的基因也是有 n 個單位，但此 n 個單位是由原來母細胞中 $2n$ 個單位隨意挑選出來的！若以 X_n 表示細胞分裂 n 次後某一個子細胞中突變單位的個數，則 $X_0, X_1, \cdots\cdots$ 也是一個馬可夫鏈。讀者可以試試看把 $P(X_{n+1} = y \mid X_n = x)$ 寫出來。

$$p(x,y) = \frac{\binom{2x}{y}\binom{2n-2x}{n-y}}{\binom{2n}{n}}$$

Markov chain (BOOK CHAPTER)

- Markov chain

- Markov Random processes

Exercise I

- x=(-1.8,-1.4,-0.8,-0.4,0.1,0.4,0.8,1.2,1.6,2.0)
- y=2*tanh(x-1)
- y=?

Exercise II: Line fitting

- x=(-1.8,-1.4,-0.8,-0.4,0.1,0.4,0.8,1.2,1.6,2.0)
- y=2*tanh(x-1)
- Use model 1 to fit a line
- y=ax+b, a=?, b=?
- What is the fitting error ?

Exercise II: Quadratic curve fitting

- x=(-1.8,-1.4,-0.8,-0.4,0.1,0.4,0.8,1.2,1.6,2.0)
- y=2*tanh(x-1)
- Use model 1 to fit a quadratic polynomial
- $y=ax^2+bx+c$, a=?, b=?, c=?
- What is the fitting error ?

Exercise III: One tanh

- $x=(-1.8,-1.4,-0.8,-0.4,0.1,0.4,0.8,1.2,1.6,2.0)$
- $y=2*\tanh(x-1)$
- Use model 2 to fit a hyper-tangent function
- $y=a*\tanh(bx+c)$, a=?, b=?, c=?
- What is the fitting error?
- Draw three fitting functions and points in a figure

Exercise IV data

- x=(-1.8,-1.4,-0.8,-0.4,0.1,0.4,0.8,1.2,1.6,2.0)
- y=tanh(2*x-1)+tanh(-3*x+1)
- y=?

Exercise V: One tanh function

- x=(-1.8,-1.4,-0.8,-0.4,0.1,0.4,0.8,1.2,1.6,2.0)
- y=tanh(2*x-1)+tanh(-3*x+1)
- $y=a_1*\tanh(b_1*x+c_1)$

  $a_1=?$, $b_1=?$, $c_1=?$
- What is the fitting error?

Exercise VI: Two tanh functions

- x=(-1.8,-1.4,-0.8,-0.4,0.1,0.4,0.8,1.2,1.6,2.0)
- y=tanh(2*x-1)+tanh(-3*x+1)
- $y=a_1*\tanh(b_1*x+c_1)+a_2*\tanh(b_2*x+c_2)$

  a_1=?, b_1=?, c_1=?, a_2=?, b_2=?, c_2=?
- What is the fitting error?
- Draw the fitting function and points